Physics of Materials

Dr. Thomas LaGrange

Chapter 8: Dislocation Kinetics

Masters Course PHYS-307

Fall 2025

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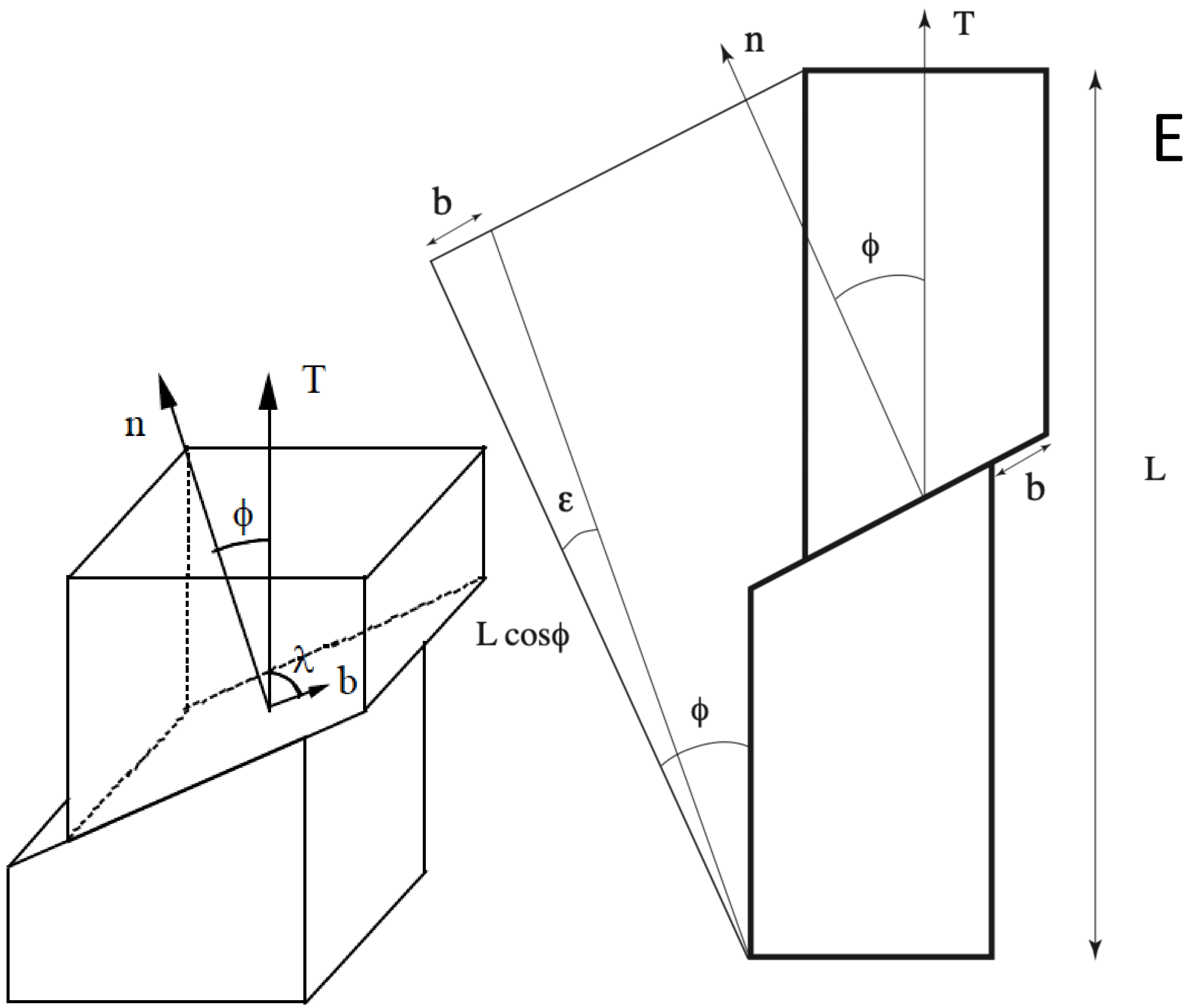
Orowan equation

$$\dot{\varepsilon} = \Lambda b \dot{u} = \Lambda b v$$



Elongation of the test-sample $\gamma = \frac{dL}{L}$

$$\varepsilon = \frac{b}{L\cos\phi} \qquad \gamma = \frac{b\cos\lambda}{L}$$

$$\gamma = \cos\lambda\cos\phi \cdot \varepsilon = m \cdot \varepsilon$$

m is the Schmid factor

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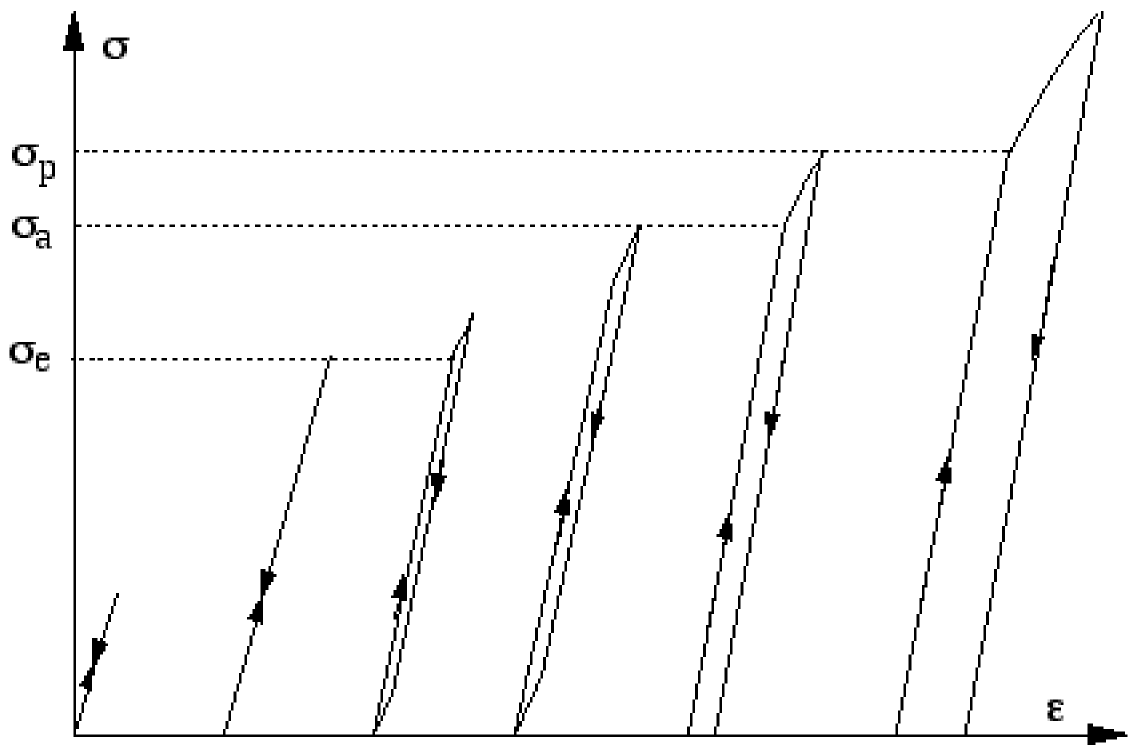
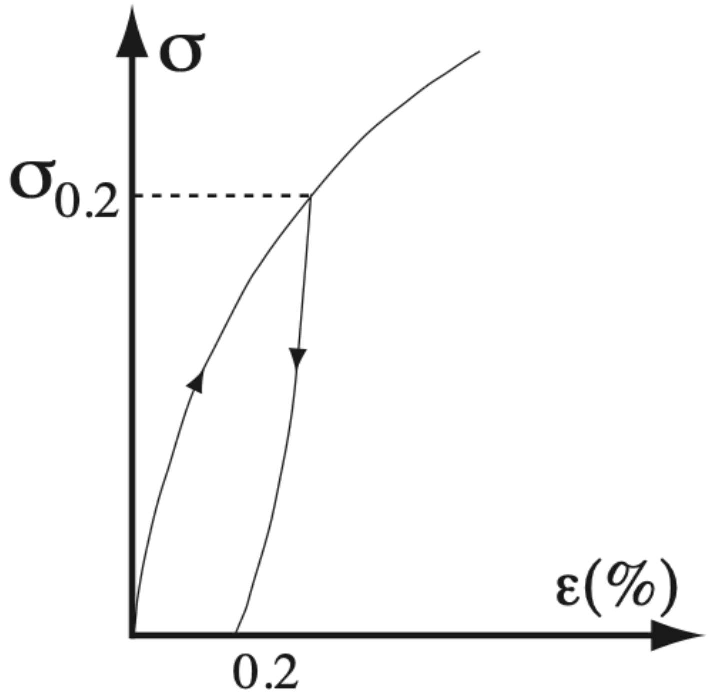
Deformation curve





Yield stress

The dislocations movement is controlled by:

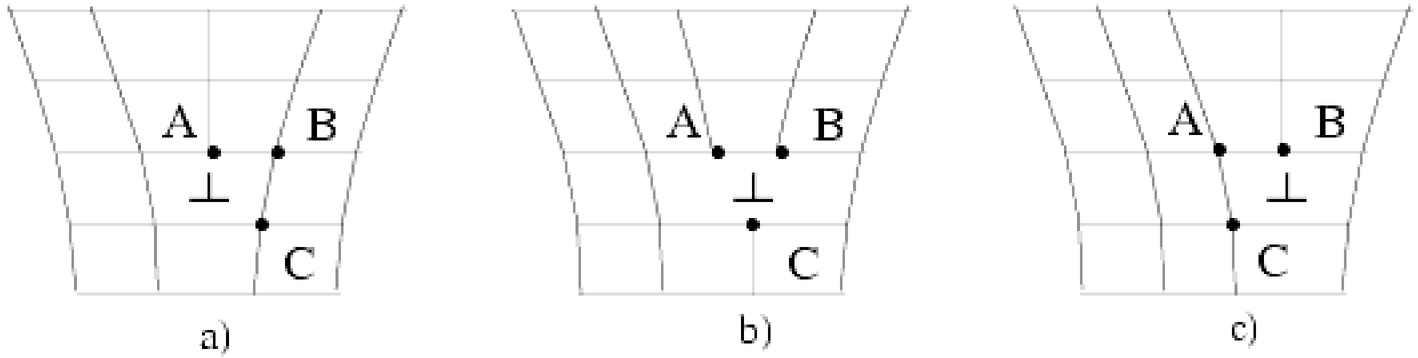
1) interactions with the lattice (Peierls-Nabarro Force)

2) interactions with other dislocations (and image forces)

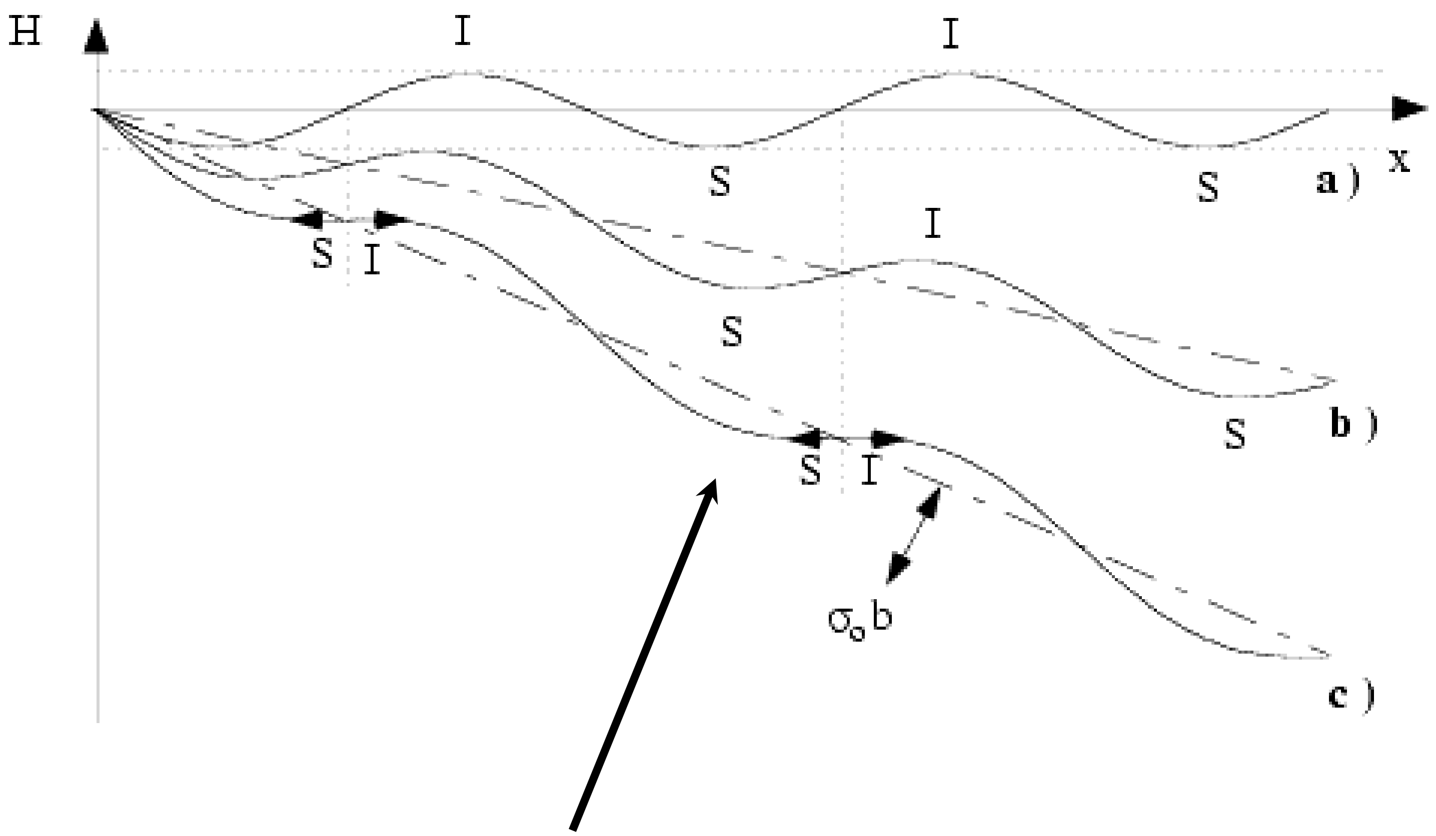
3) interaction with point defects (solute drag)

4) interactions with extended defects (precipitates, grain boundaries)

Interaction of dislocations with the crystal lattice



Free energy $\Delta G = W(x) - \sigma b x$



There is movement if:

$$\frac{\partial W(x)}{\partial x} - \sigma_0 b = 0$$

Potential

$$W(x) = -\left(\frac{\tau_I - \tau_S}{2}\right)\sin\frac{2\pi x}{a}$$

$$\frac{\partial W(x)}{\partial x} = -\frac{\pi}{a}(\tau_I - \tau_S)\cos\frac{2\pi x}{a}$$

Horizontal tangent: $\frac{2\pi x}{a} = \pi$ $\quad \sigma_0 b = \frac{\pi}{a}(\tau_I - \tau_S)$ $\quad \sigma_0 = \frac{\pi}{ab}(\tau_I - \tau_S)$

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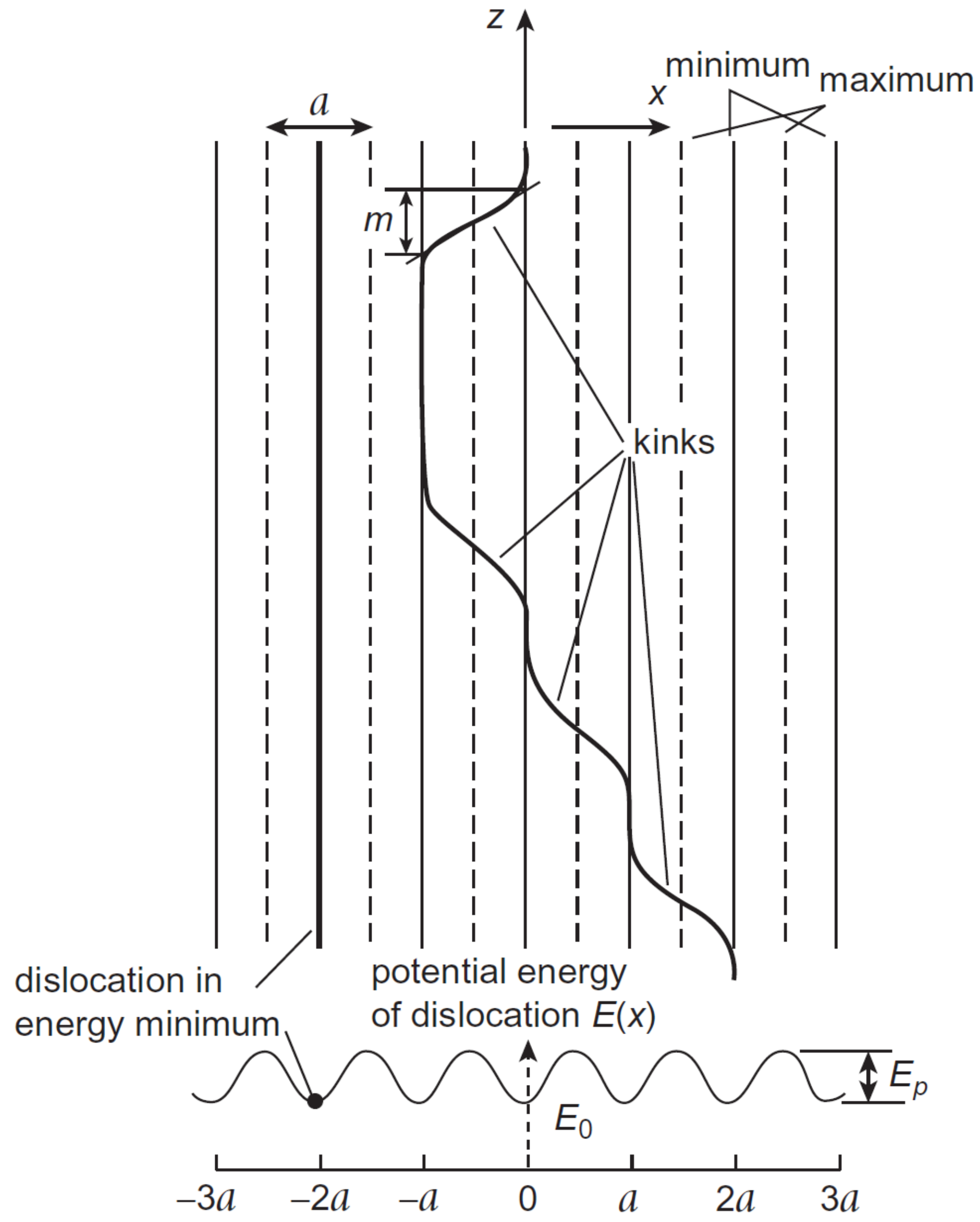
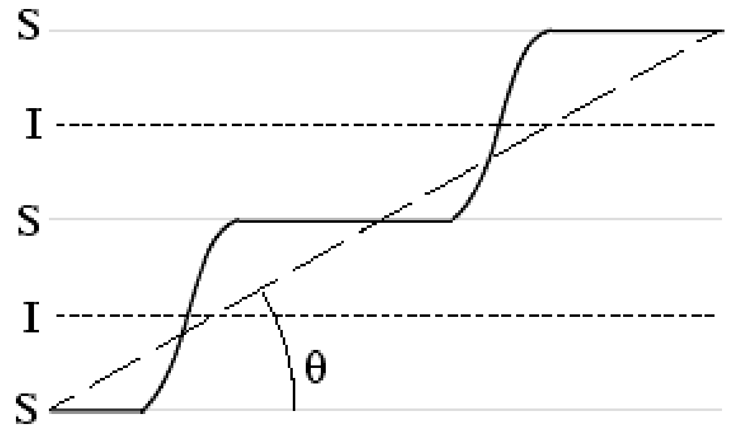
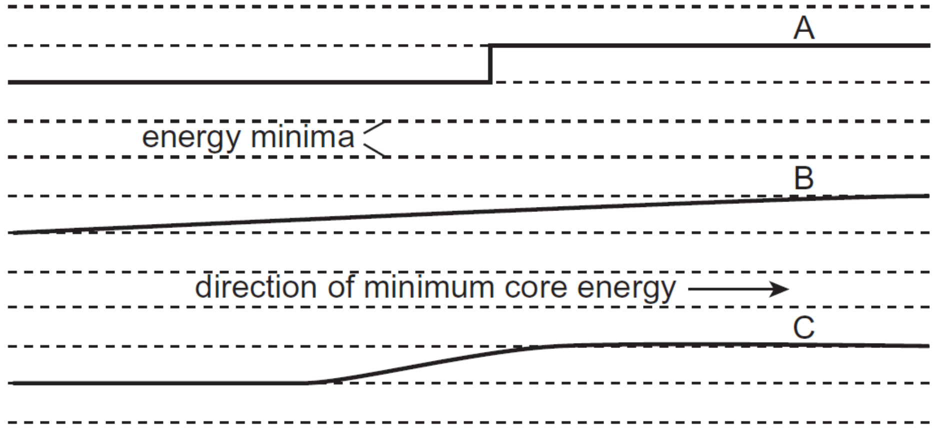
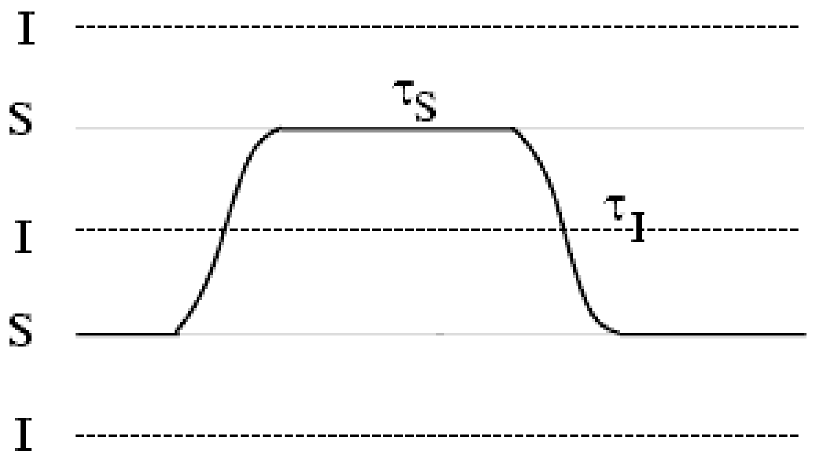
Kinks







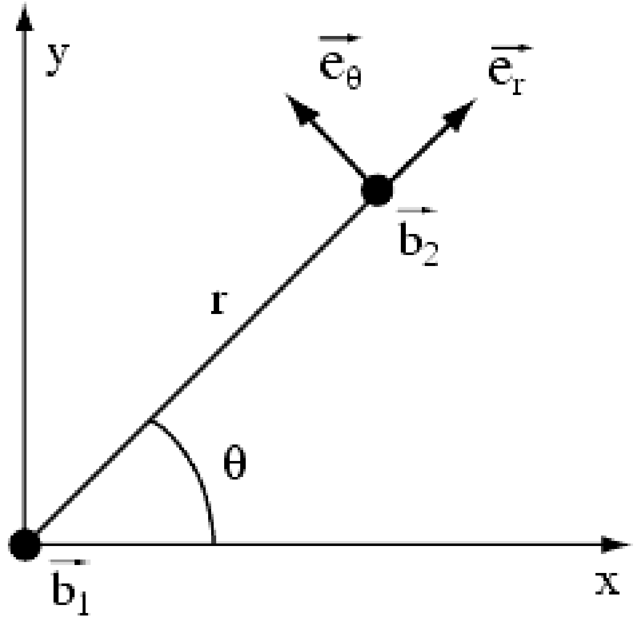


Double Kink mechanism

We get a Peierls stress of order $\frac{\mu}{1000}$

Interaction between dislocations

Two parallel screw dislocations



$$\vec{b_1} = (0,0,b_1)$$

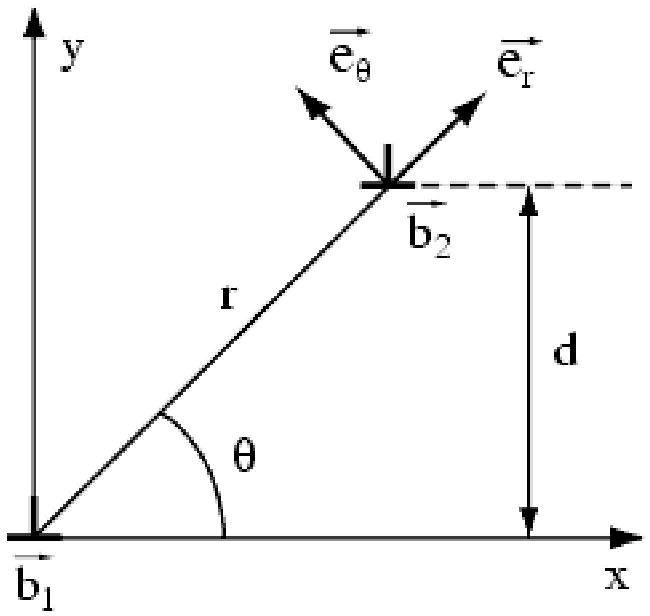
$$\vec{b_2} = (0,0,b_2)$$

$$\sigma_{\theta z} = \frac{\mu b_1}{2\pi r}$$

$$F_r = b_2 \sigma_{\theta z} = \frac{\mu b_1 b_2}{2\pi r} = \pm \frac{\mu b^2}{2\pi r}$$

Two parallel edge dislocations

$$\sigma_{xy} = D \frac{\cos\theta \cos 2\theta}{r}$$



$$\vec{b_1} = (b_1,0,0,)$$

$$\vec{b_1} = (b_2,0,0,)$$

$$D = \frac{\mu b}{2\pi(1-\nu)}$$

$$\sigma_{xx} = -D \frac{\sin\theta(2 + \cos 2\theta)}{r}$$

$$F_x = b_2 \sigma_{xy}$$

$$F_x = \frac{\mu b_1 b_2}{2\pi(1-\nu)} \frac{\cos\theta \cos 2\theta}{r}$$

$$F_y = -b_2 \sigma_{xx}$$

$$F_y = \frac{\mu b_1 b_2}{2\pi(1-\nu)} \frac{\sin\theta(2 + 2\cos 2\theta)}{r}$$

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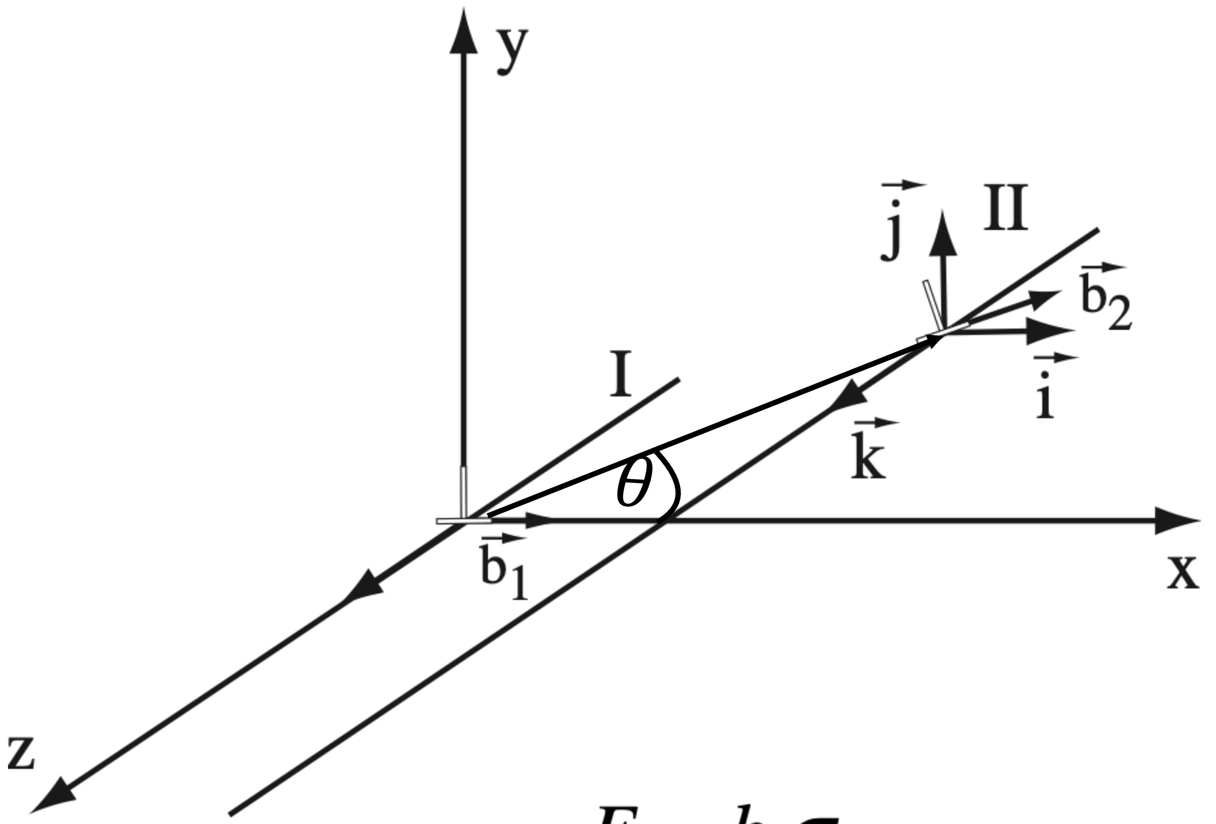
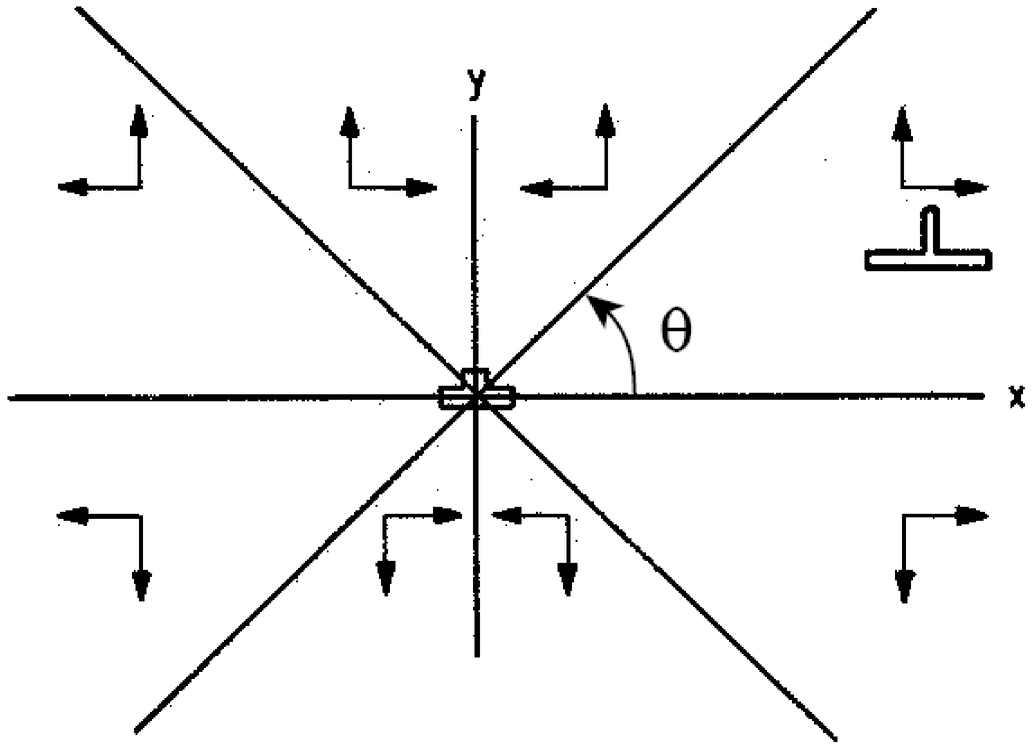
Solution of the problem:
2 edge dislocations interaction



$$F_x = b_2 \sigma_{xy}$$



$$F_y = -b_2 \sigma_{xx}$$

$$f_x = \frac{\mu b_1 b}{2\pi(1-\nu)} \cdot \frac{\cos\theta \, \cos 2\theta}{r} \quad \text{and} \quad f_y = \frac{\mu b_1 b}{2\pi(1-\nu)} \cdot \frac{\sin\theta \, (2+\cos 2\theta)}{r}$$

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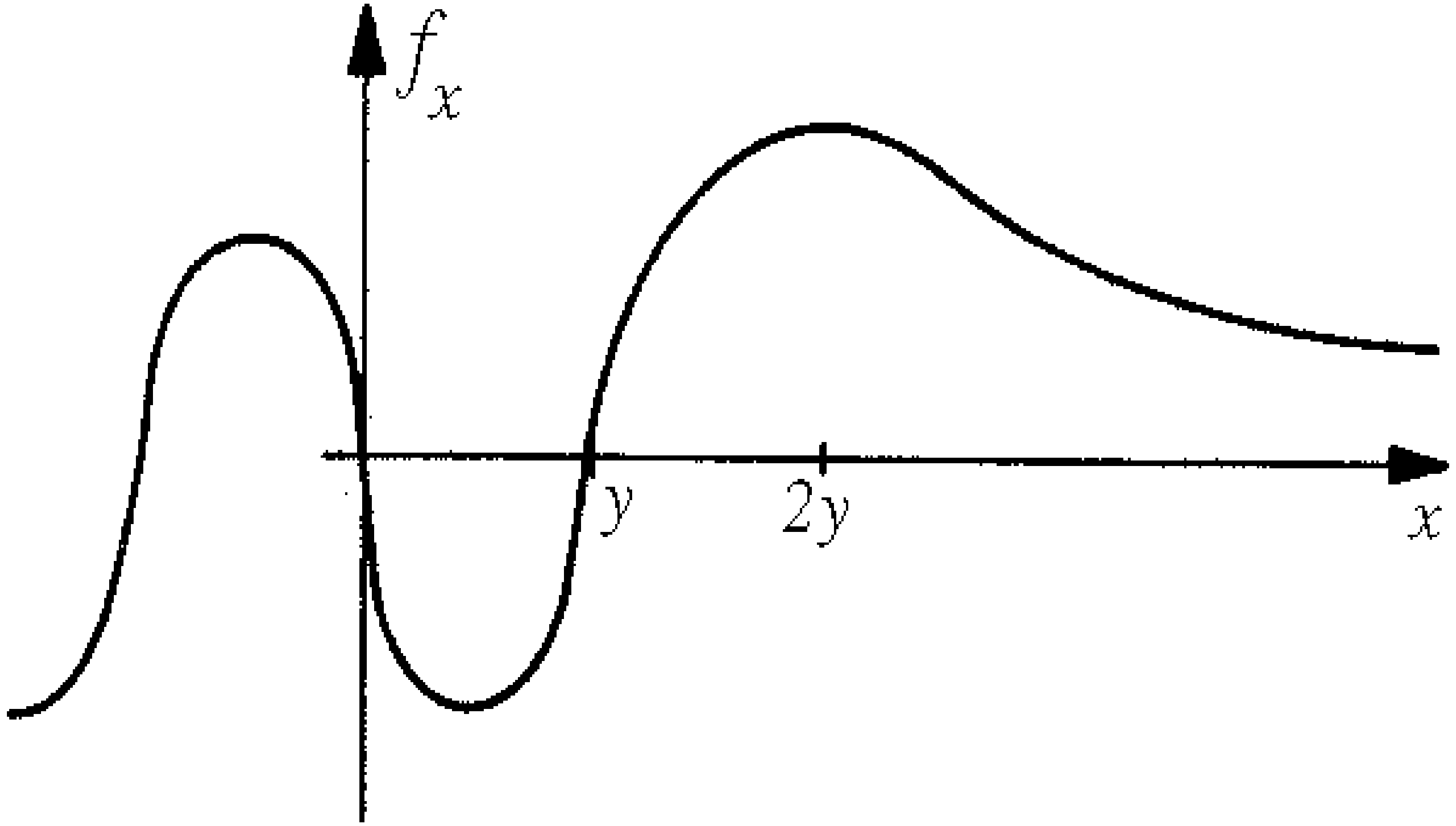
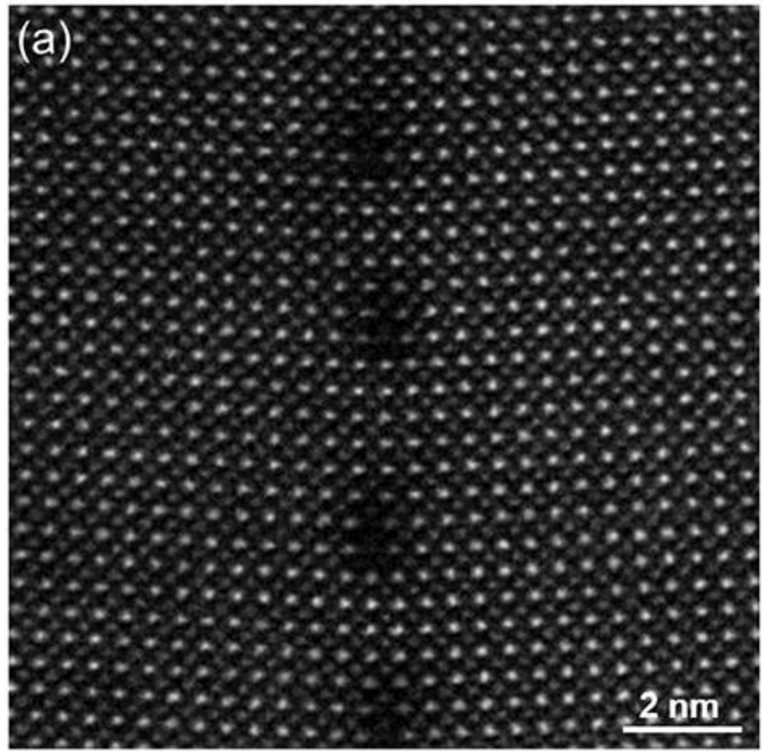
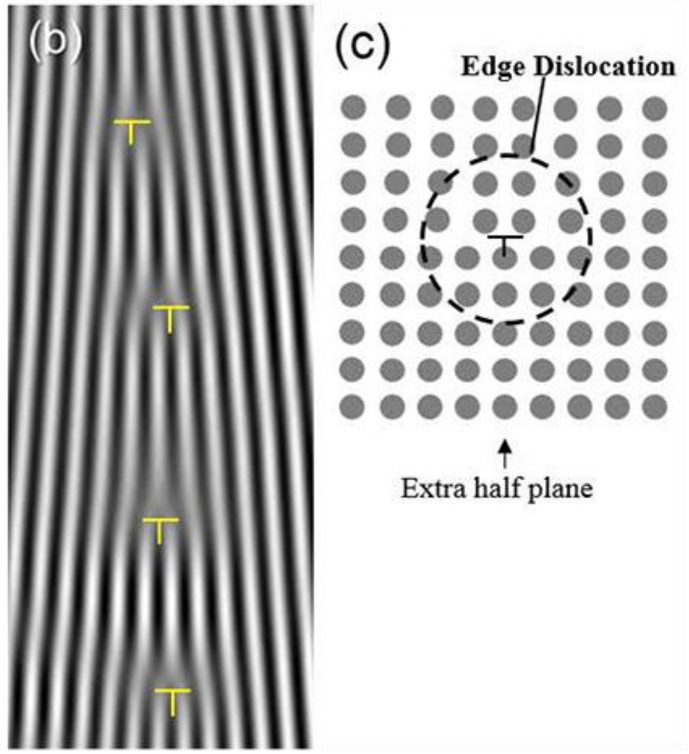
The Peach Koehler Force between edges dislocation explains small angle grain boundary formation



$$f_x = \frac{\mu b_1 b}{2\pi(1-\nu)} \frac{x(x^2 - y^2)}{(x^2 + y^2)^2}$$





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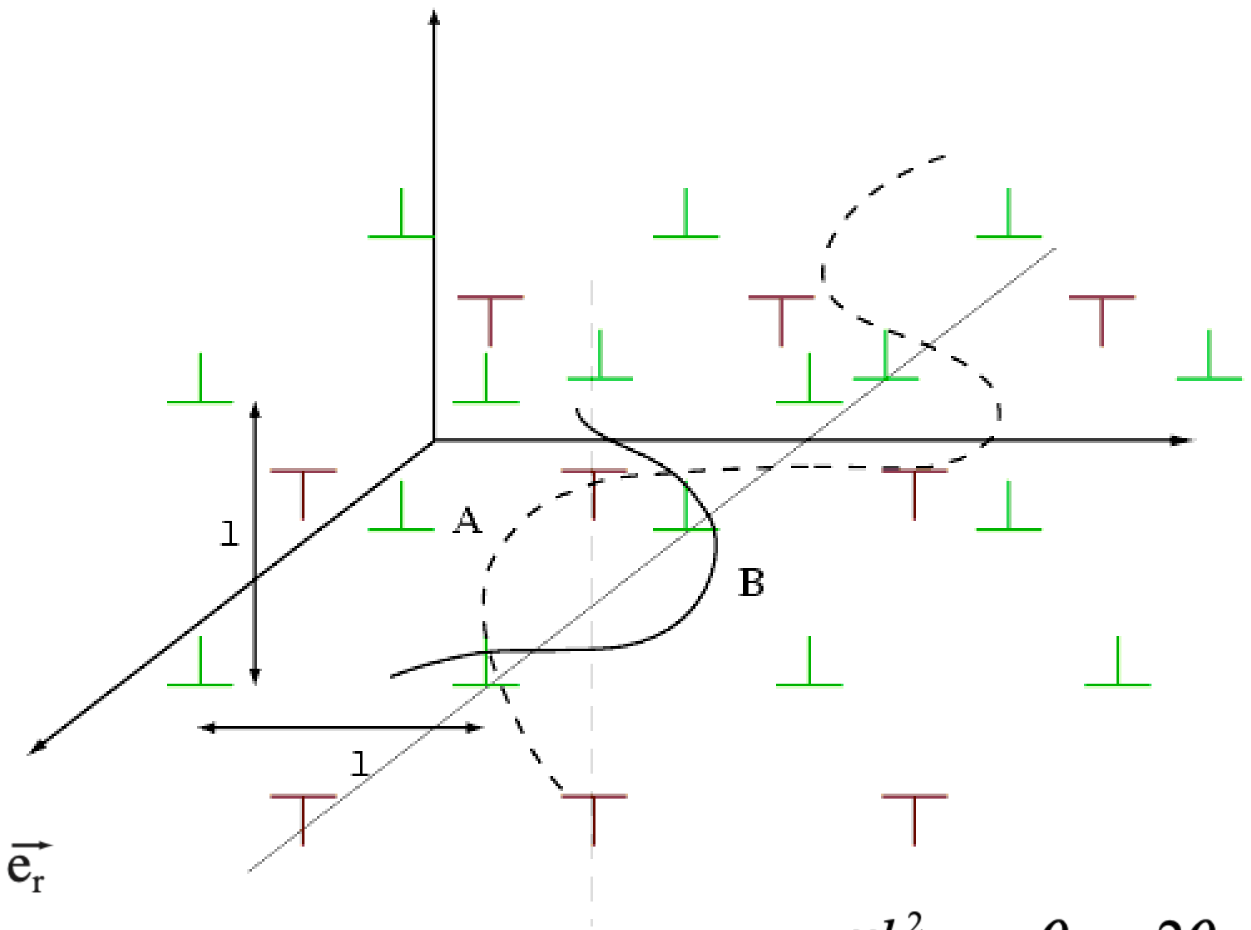
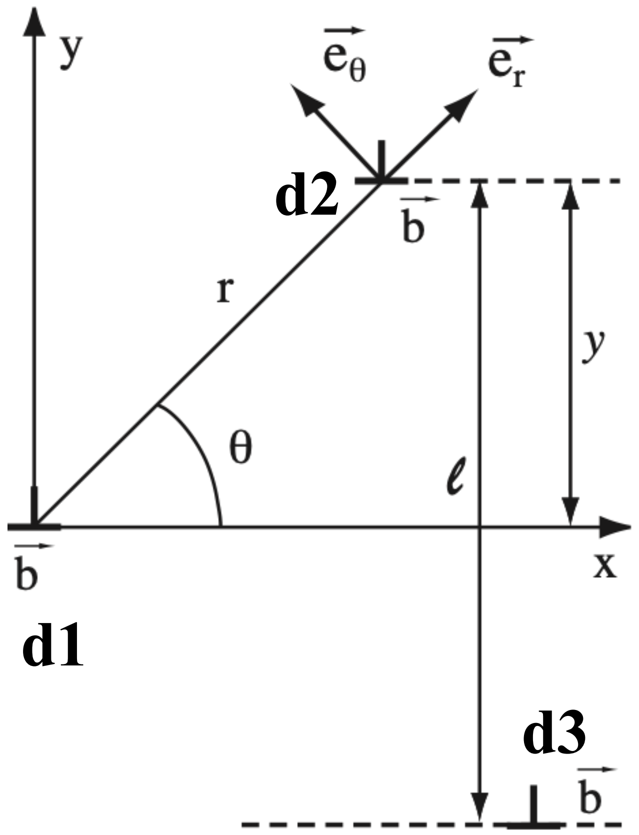
Dislocations: long distance interaction



Frank Network

e.g., contains parallel edge dislocation lying on parallel (111) glide planes (green same $\vec{b}$ and black opposite $-\vec{b}$)



$$\sigma_{xy}(d2,d3 \rightarrow d1) = -\frac{\mu b^2}{2\pi K}\frac{\cos\theta\cos 2\theta}{r} = -\frac{\mu b^2}{4\pi K}\frac{\sin 2\theta\cos 2\theta}{y} = -\frac{\mu b^2}{8\pi K}\frac{\sin 4\theta}{y}$$

$$\sigma_{max}\left(\frac{\pi}{8}\right) = -\frac{\mu b^2}{8\pi K}\left(\frac{1}{y}+\frac{1}{\ell - y}\right) = -\frac{\mu b^2}{8\pi K \ell}\left(\frac{\ell}{y(1-y/\ell)}\right) \qquad K = \frac{1-\nu}{1-\nu\cos^2\psi}$$

$$Y = \frac{y}{\ell} \qquad \sigma_{max} = -\frac{\mu b^2}{2\pi K \ell}\left(\frac{1}{4Y(1-Y)}\right) = -\frac{\mu b^2}{2\pi K \ell} f(Y)$$

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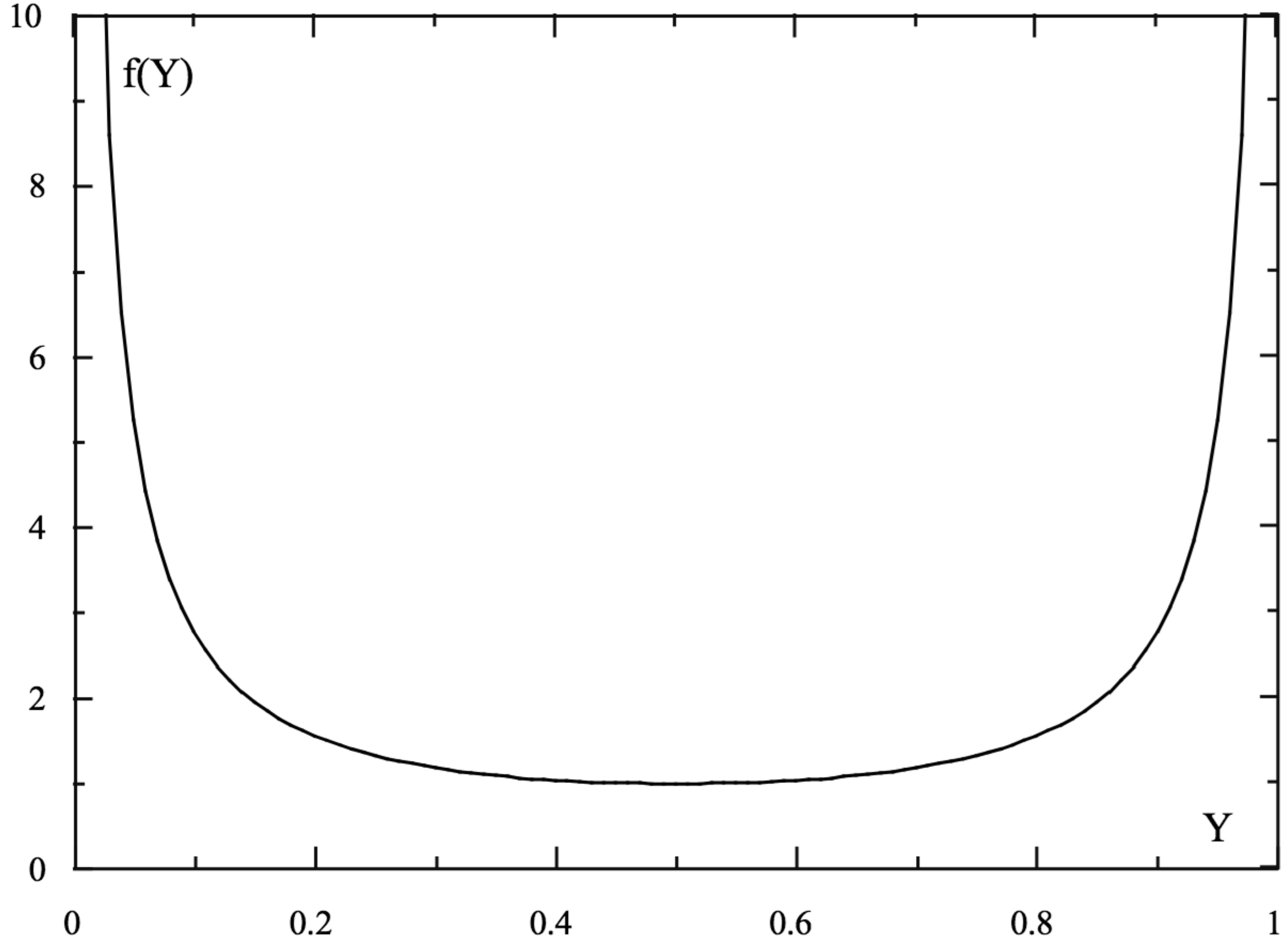
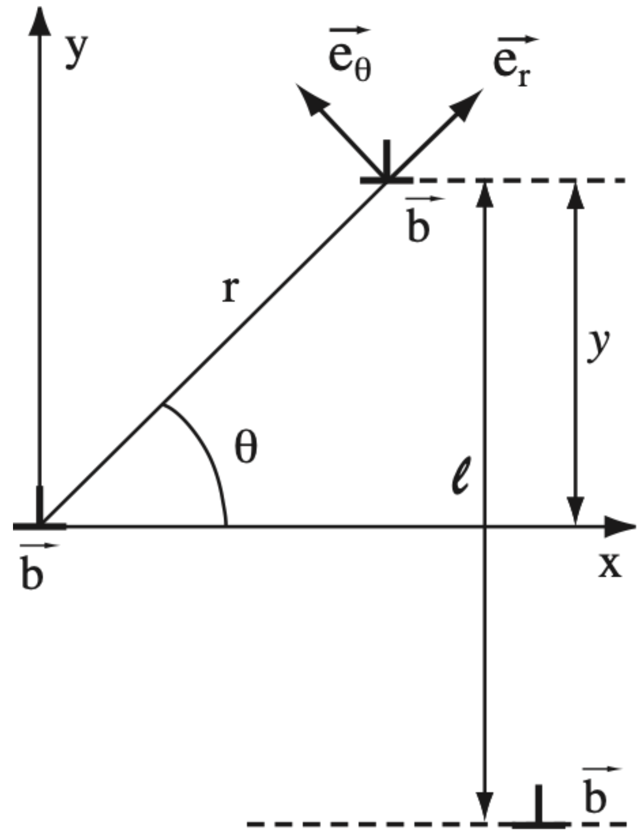
Dislocations: long distance interaction





$$\sigma_{max} = -\frac{\mu b^2}{2\pi K \ell}\left(\frac{1}{4Y(1-Y)}\right) = -\frac{\mu b^2}{2\pi K \ell} f(Y) \qquad Y = \frac{y}{\ell}$$

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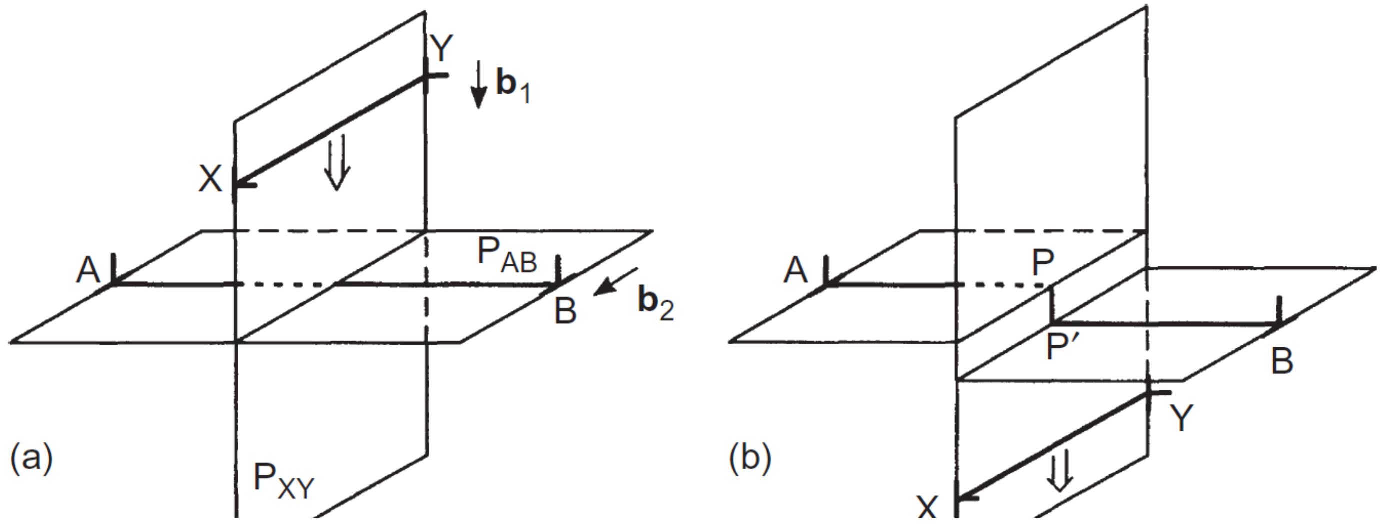
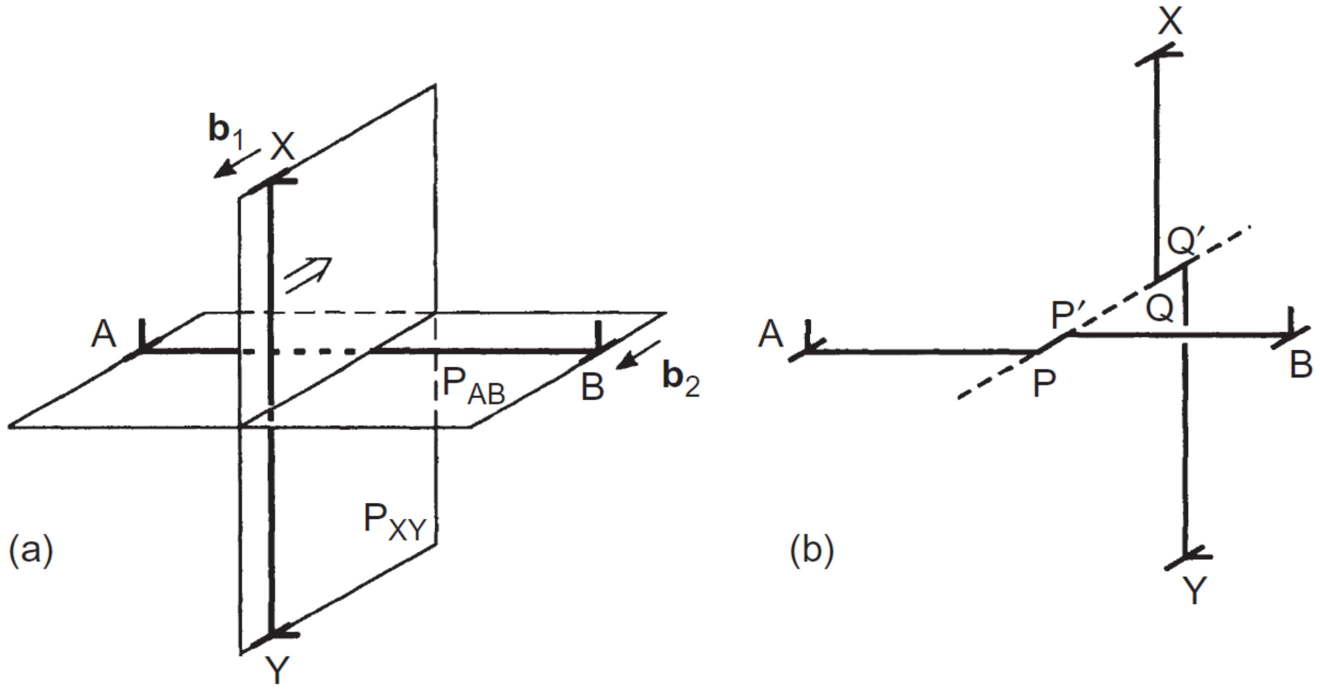
Dislocations: short distance interaction “Jogs”

edge-edge type





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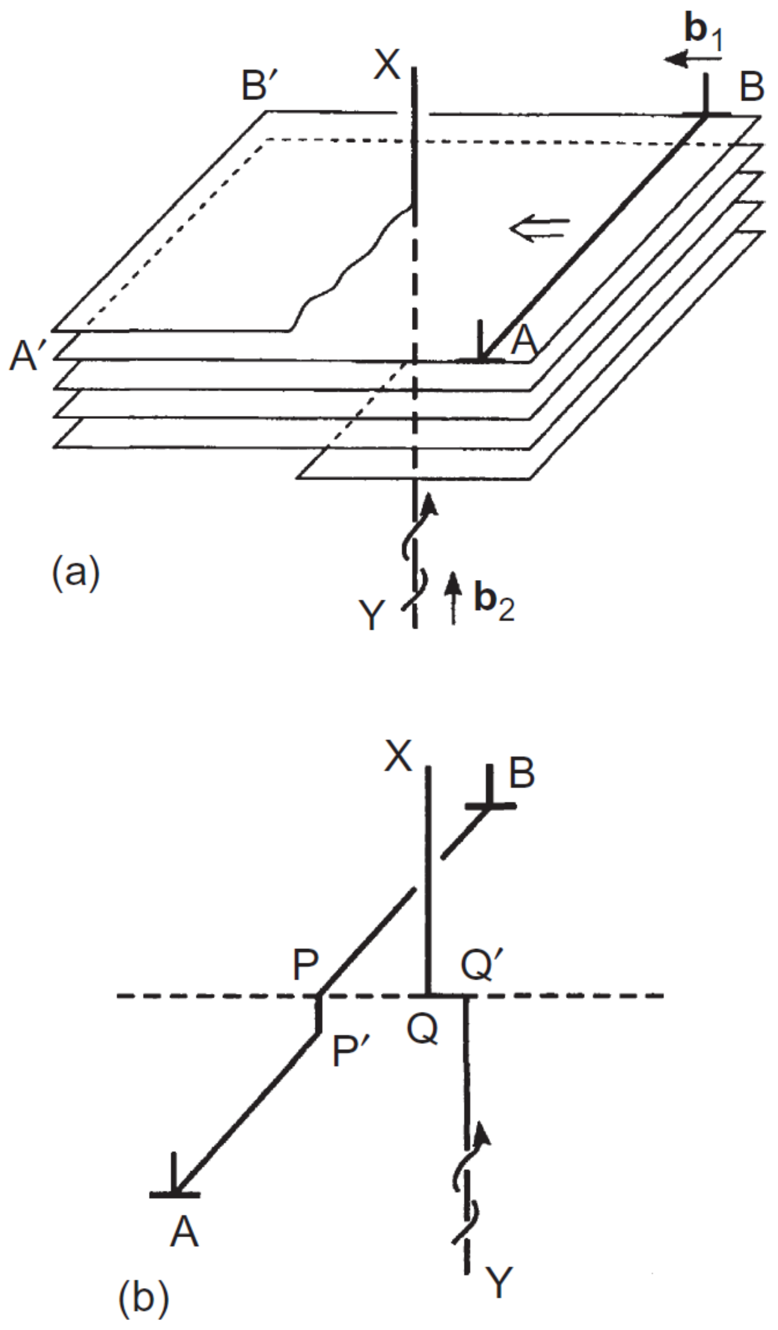
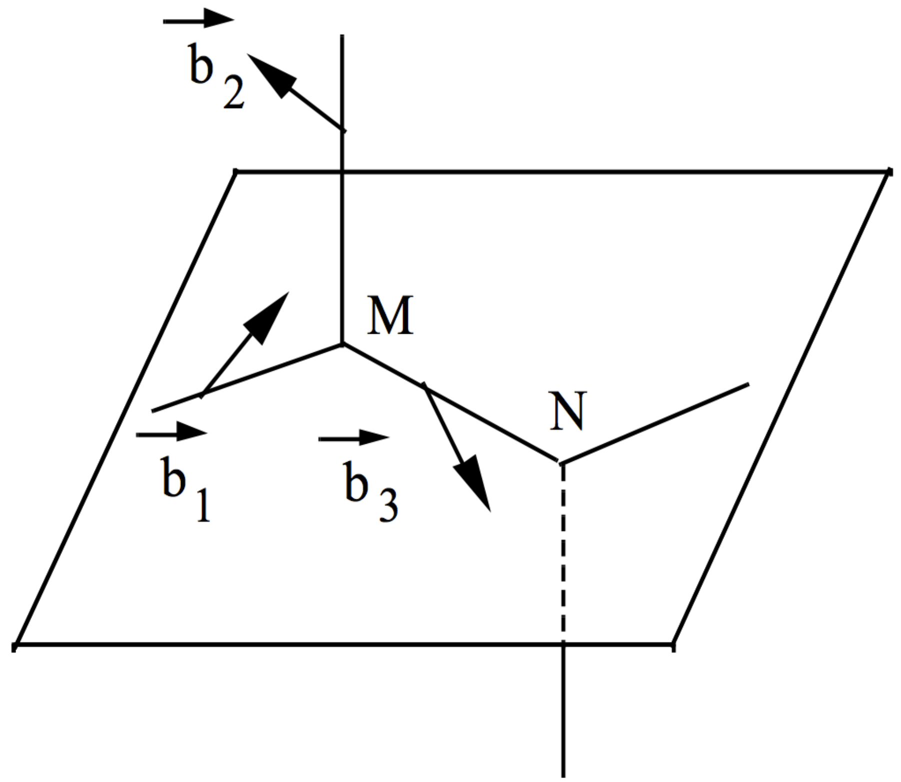
Dislocations: short distance interaction

edge-screw



tree



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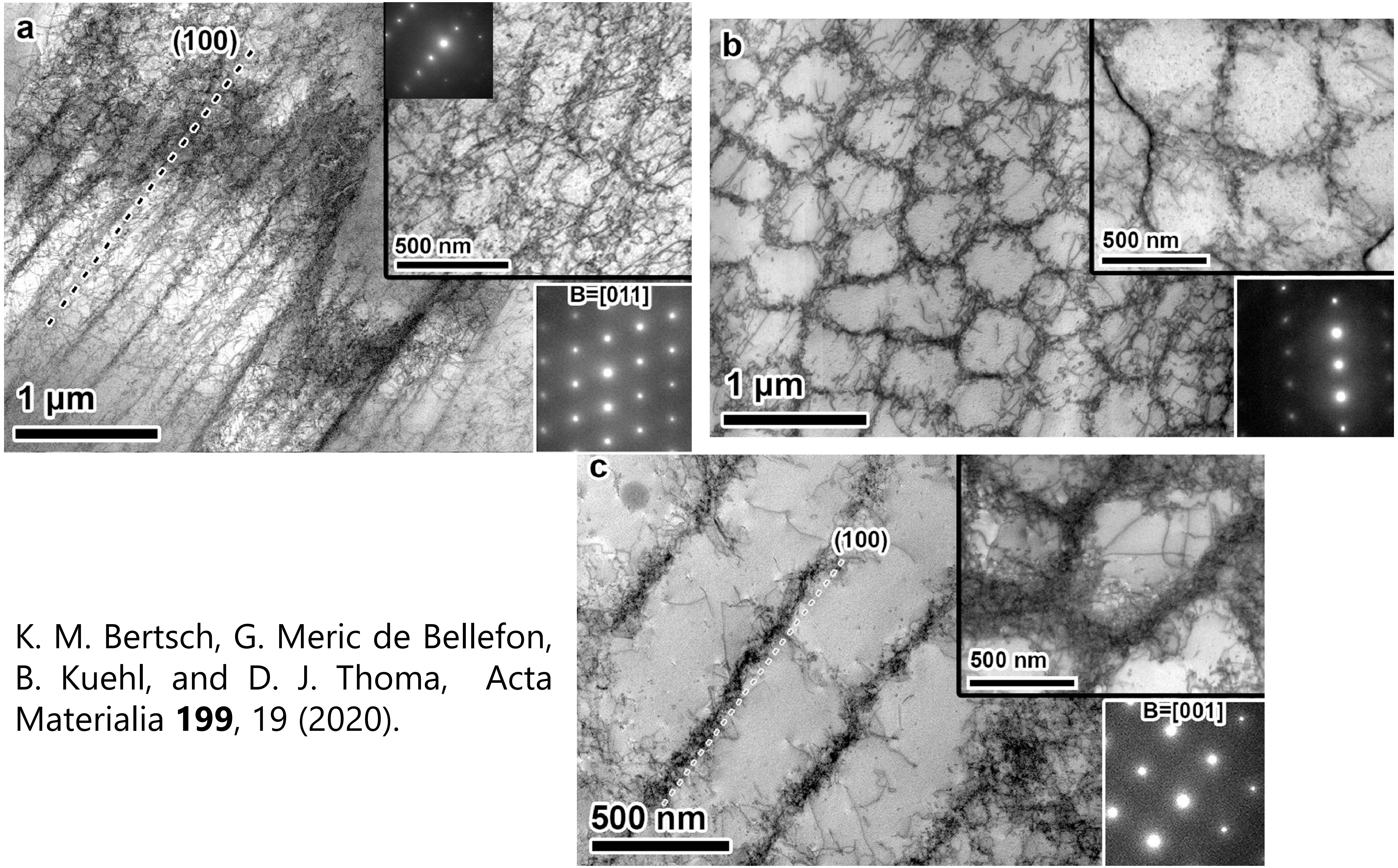
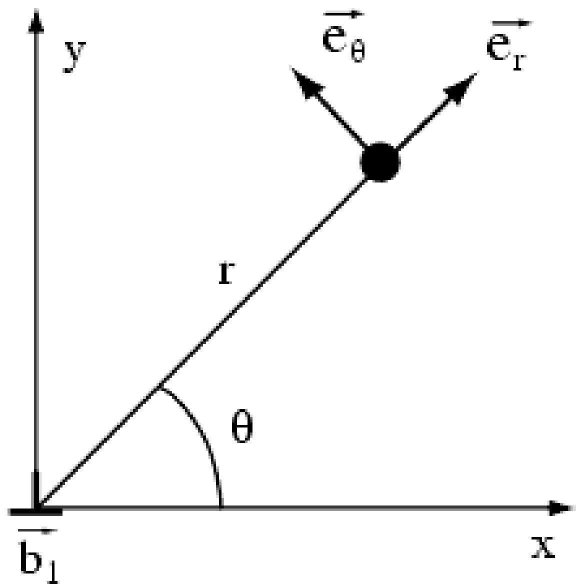
Forests dislocations and Cells



K. M. Bertsch, G. Meric de Bellefon, B. Kuehl, and D. J. Thoma, Acta Materialia **199**, 19 (2020).

Elastic interaction with point defects



$$\sigma_p = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}) = \frac{1}{3}Tr(\sigma)$$

$$\Delta\Omega = \eta\Omega$$

Hydrostatic pressure exerted by the dislocation on the impurity:

$$p = \frac{1}{3}\sum_i \sigma_{ii} = -\frac{\mu b}{3\pi}\frac{(1+\nu)}{(1-\nu)}\frac{\sin\theta}{r}$$

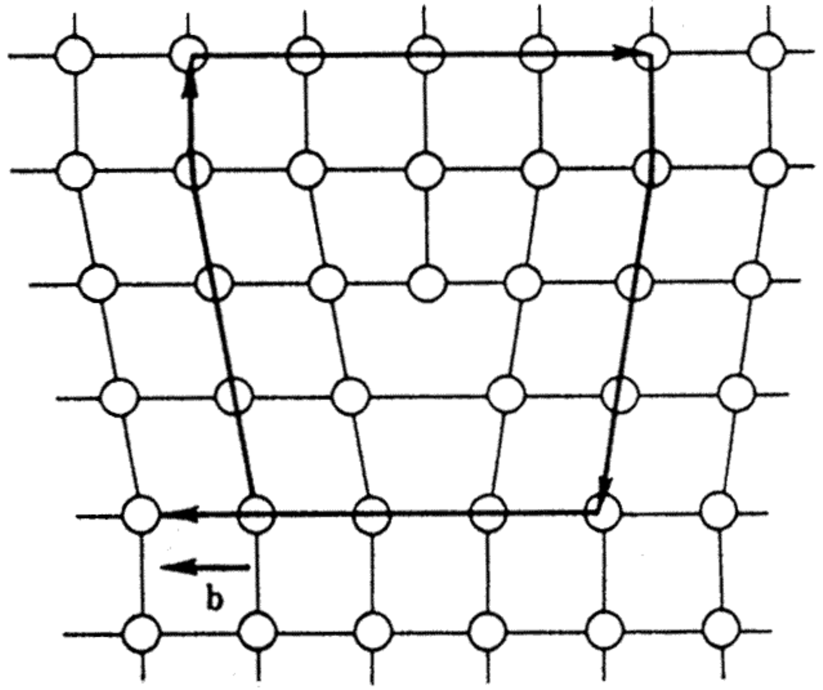
Energy variation from infinite distance:

$$W_\infty - W_I = -p\eta\Omega = \eta\Omega\frac{\mu b}{3\pi}\frac{(1+\nu)}{(1-\nu)}\frac{\sin\theta}{r}$$

$$\eta < 0 \Rightarrow \sin\theta > 0$$

$$\eta > 0 \Rightarrow \sin\theta < 0$$

lower energy



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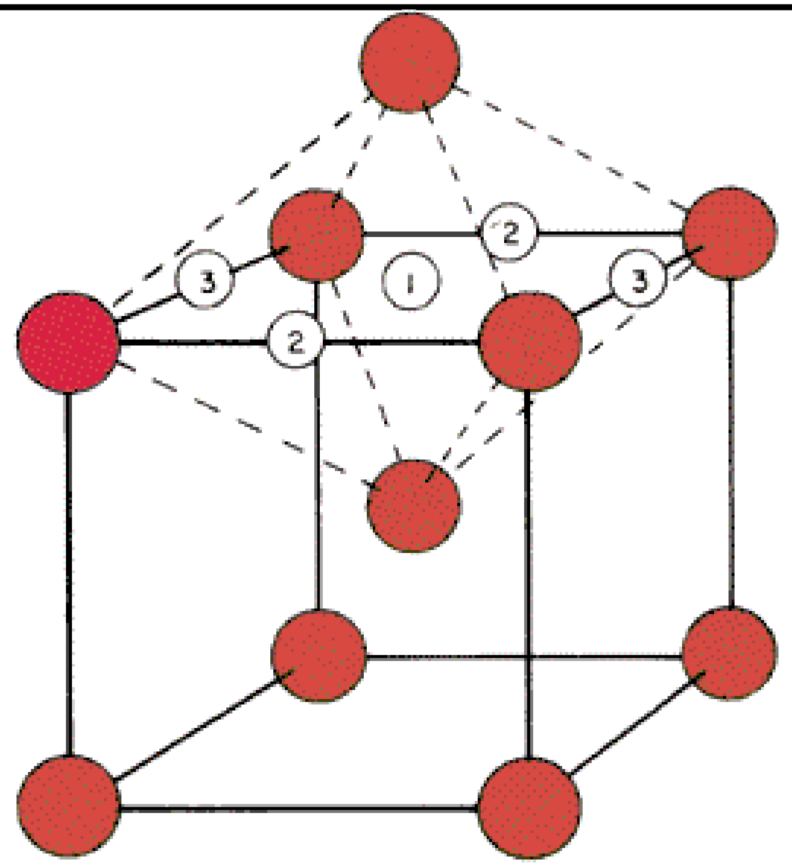
Tetrahedral effect: shear-stress force in BCC



Effect on screw dislocations
e.g., ferritic steel and interaction with interstitial carbon in bcc Fe lattice

$$W_{tetragonal} > W_{size} > W_{module} > W_{electrostatic}$$

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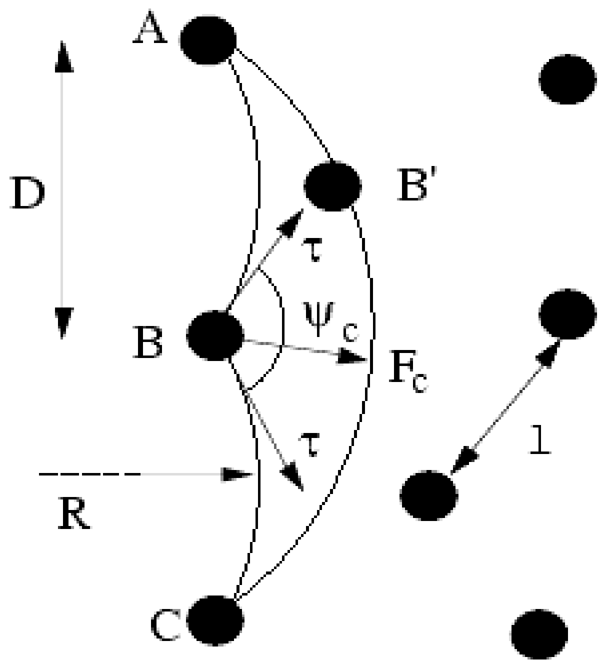
Movement of the dislocation through a distribution of point defects

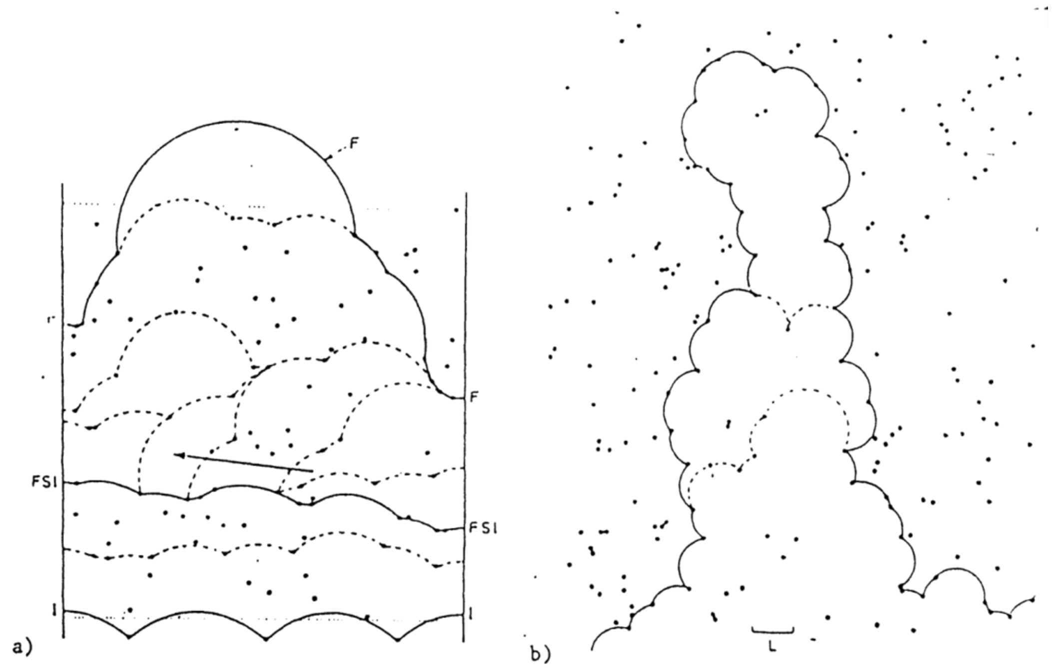


$$R = \frac{\tau}{\sigma b}$$

$$F_c = 2\tau \cos\frac{\psi}{2} = \beta\tau \quad 0 < \beta < 2$$

$\beta \approx 0 \Rightarrow \psi \approx \pi$ weak obstacles

$\beta \approx 2 \Rightarrow \psi \approx 0$ strong obstacles



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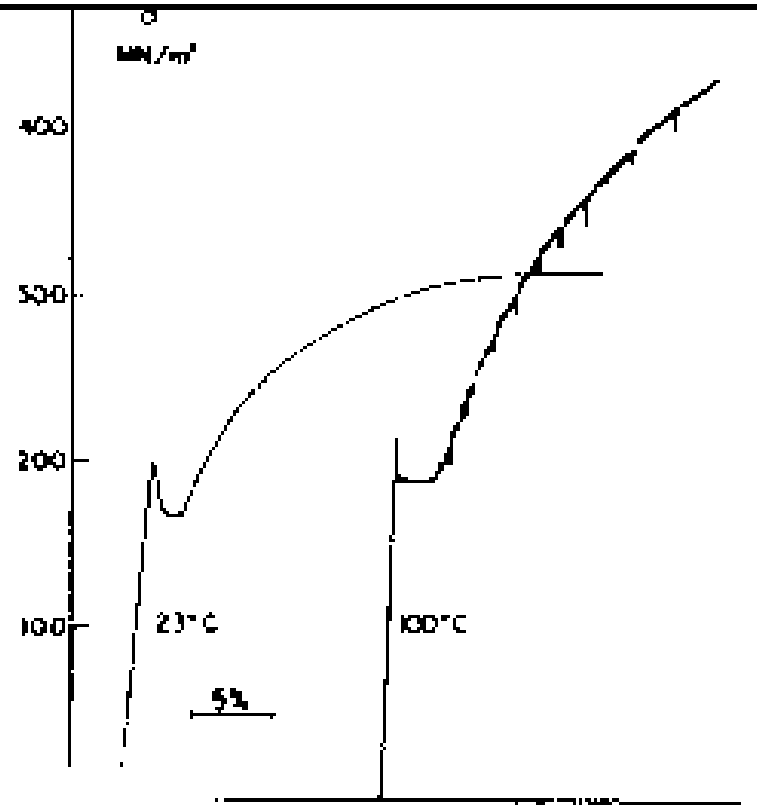
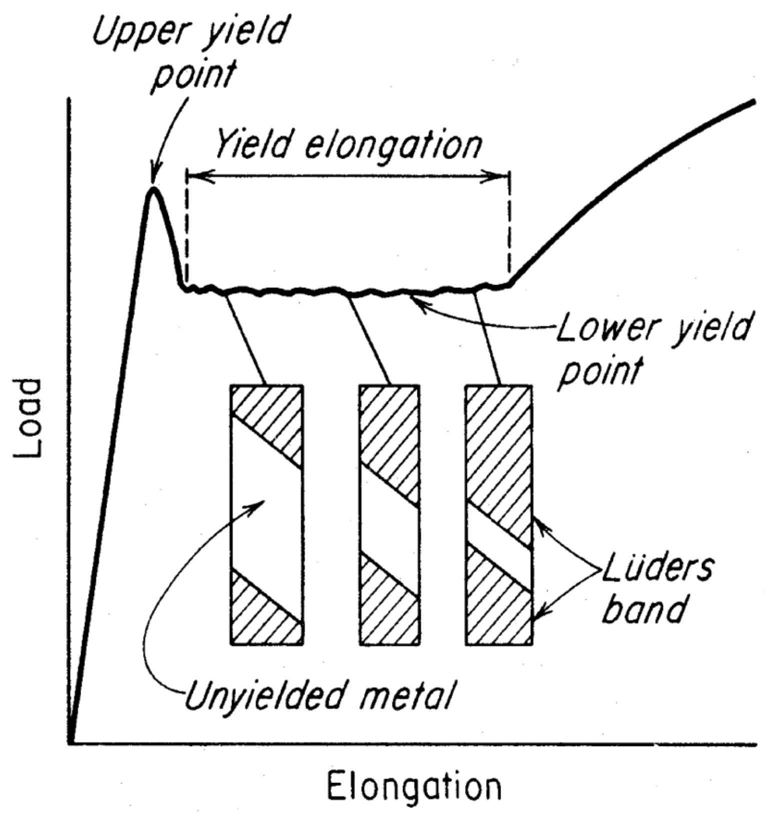
Instabilities in deformation curves



Piobert - Lüders

Slip band propagation due dislocation multiplication followed by hardening due to interaction with point defects (e.g., carbon atoms in steel)



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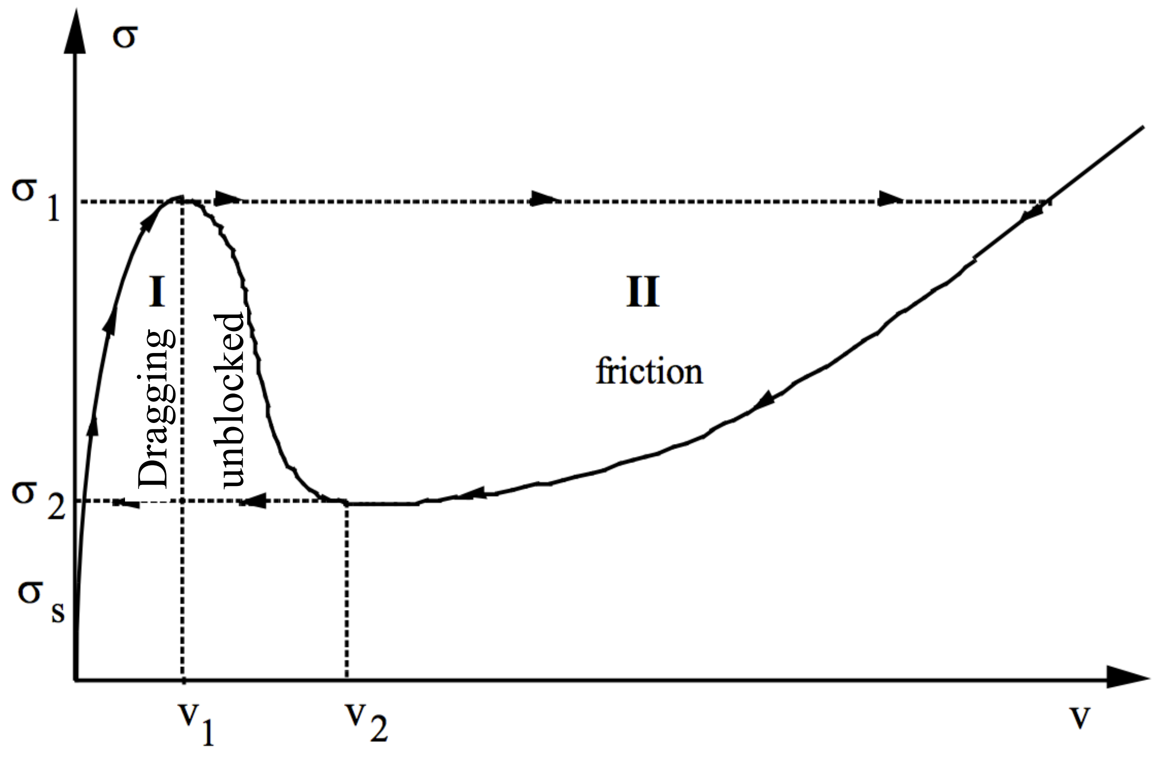
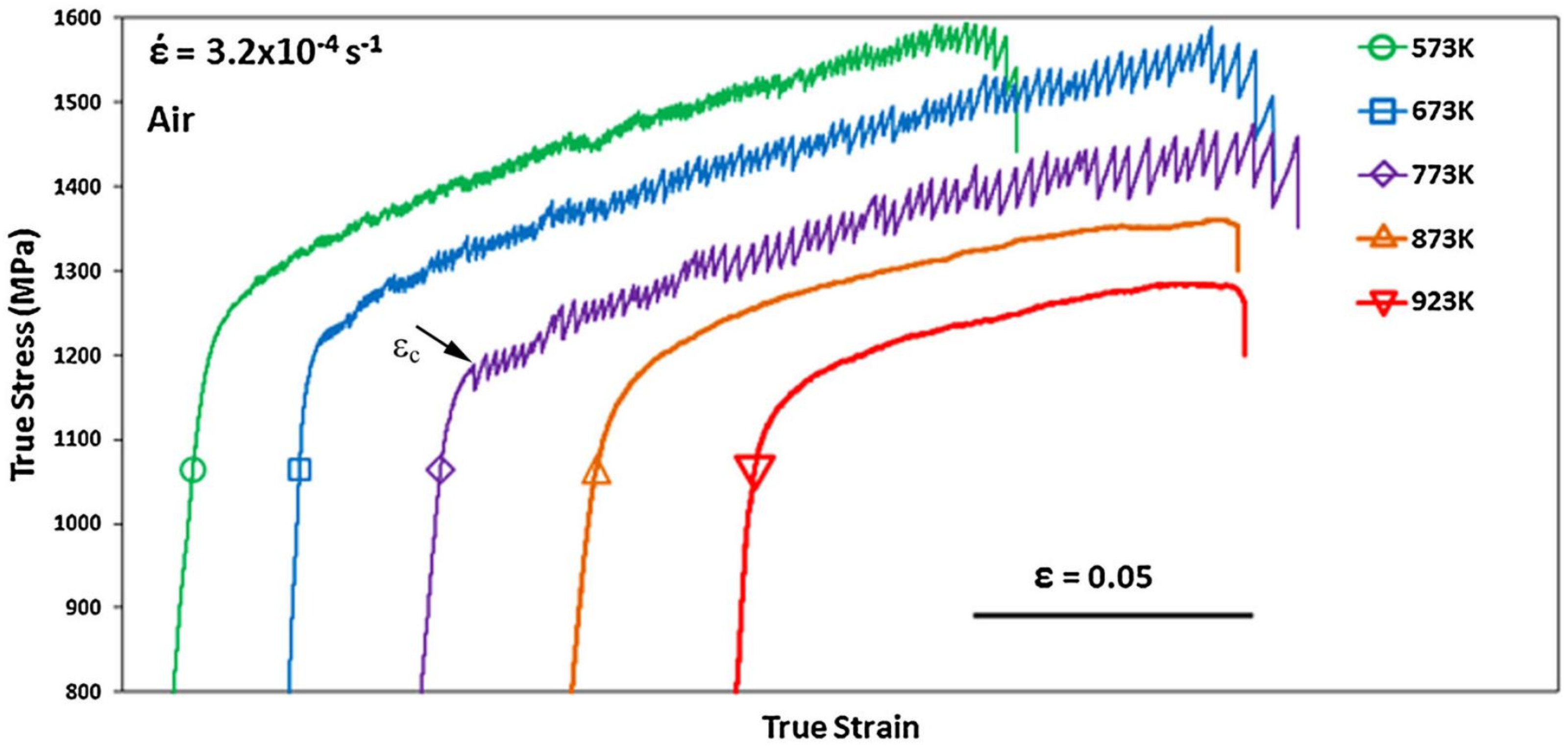
Instabilities in deformation curves



Portevin - Le Châtelier effect

Instabilities in the flow stress due to pinning and depinning of dislocations from point defects (e.g., diffusion (pipe) of solute atoms to the dislocation core) that change their velocities



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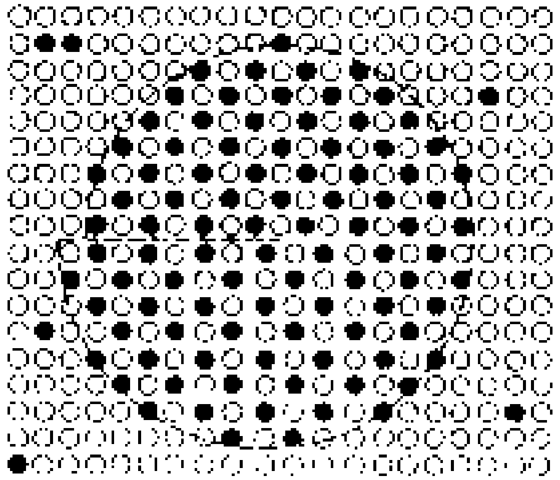
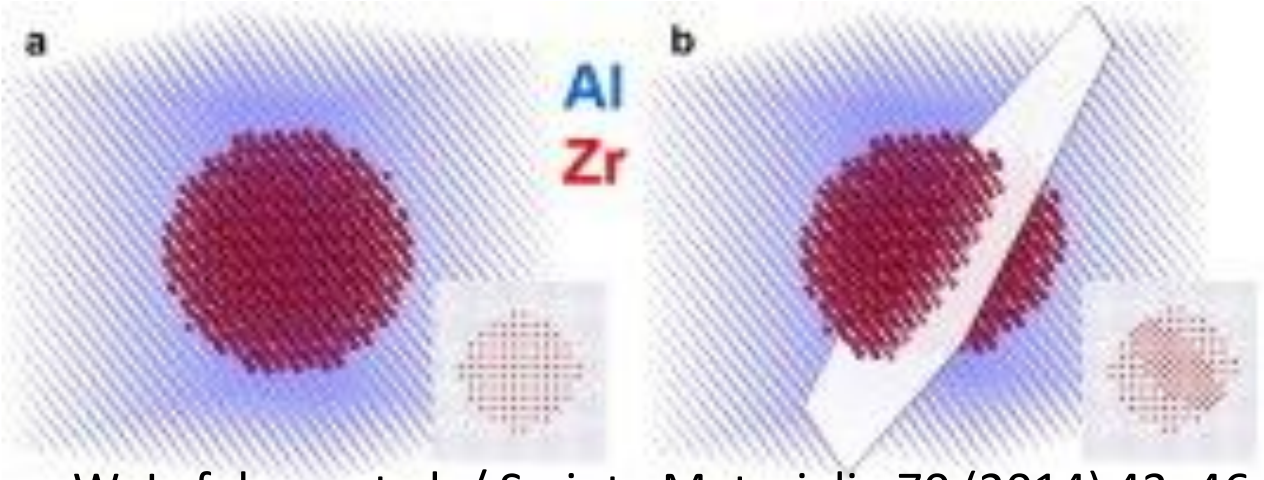
Interactions with precipitates

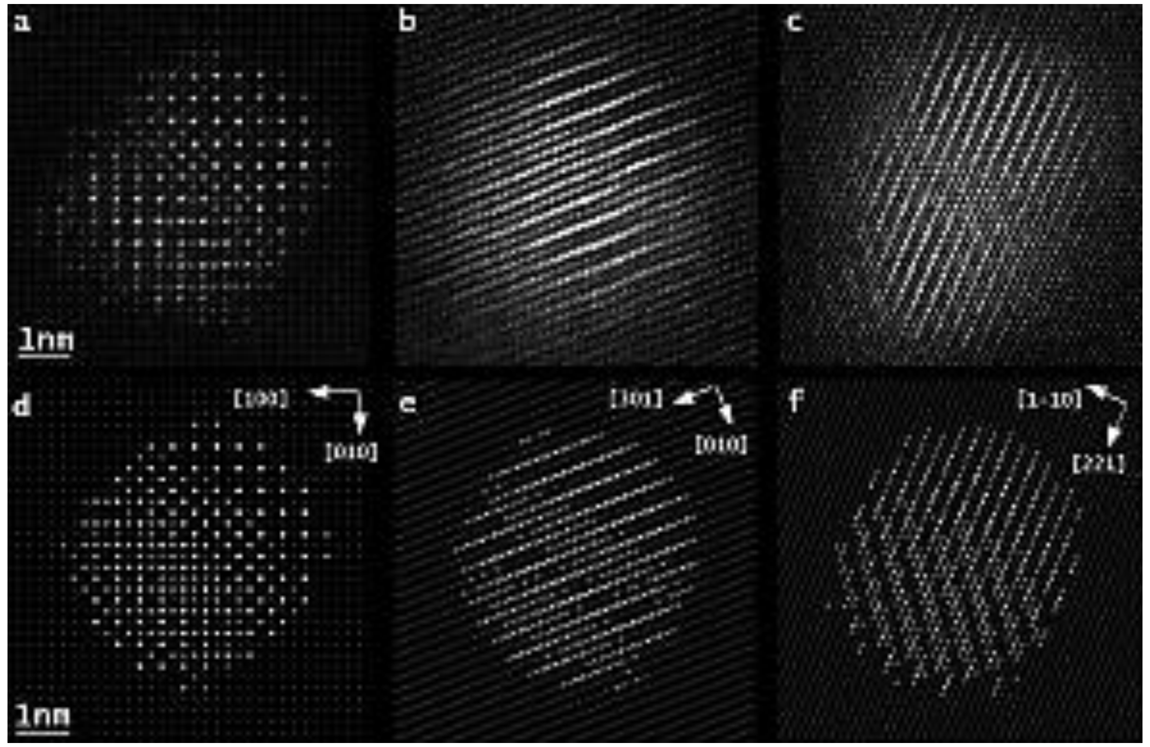
Cutting



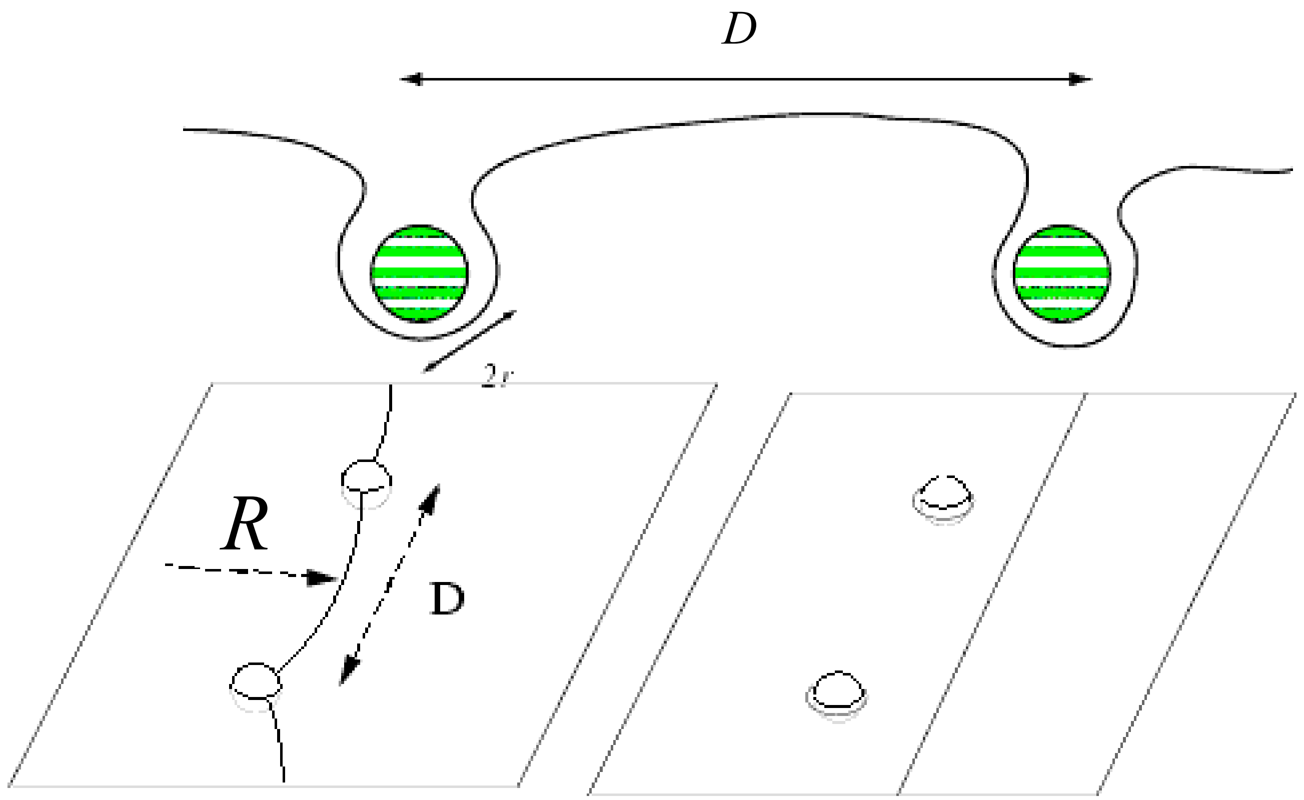
$$\sigma_c \sim \frac{\alpha \gamma_P^{2/3} f^{1/2} r^{1/2}}{\mu^{1/2} b^2}$$



W. Lefebvre et al. / Scripta Materialia 70 (2014) 43–46



Bypass



$$\tau \approx \frac{1}{2}\mu b^2$$

$$\sigma_c \approx \frac{\tau}{rb} \approx \frac{\mu b}{2r}$$

$$\sigma_c \approx \frac{\tau}{Rb} = \frac{2\tau}{Db} \approx \frac{\mu b}{D}$$

$$D \propto \frac{r}{\sqrt{f}} \Rightarrow \sigma_c \approx \frac{\mu b \sqrt{f}}{r}$$

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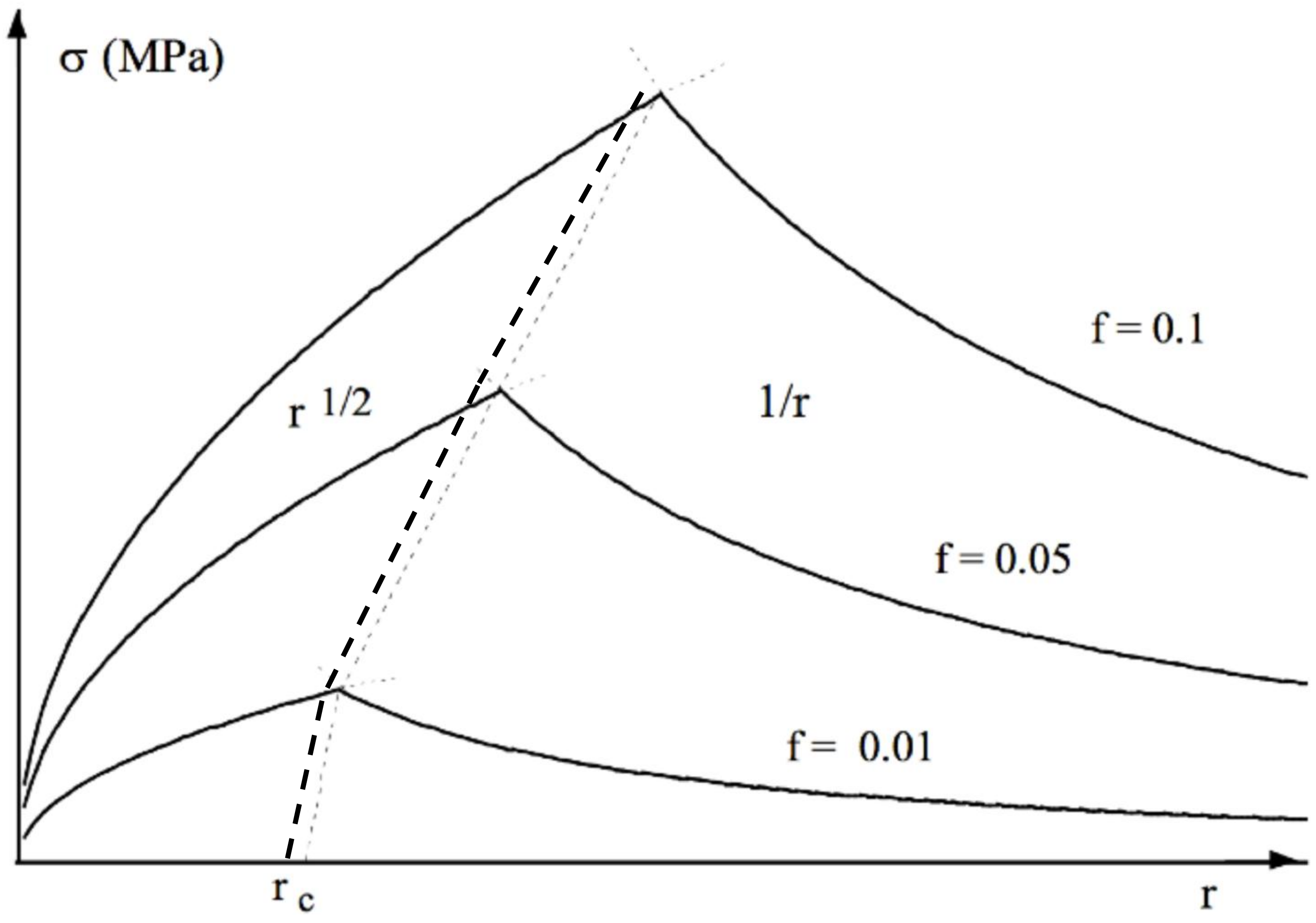
Competition cutting-bypass



$$\sigma_c \approx \frac{\alpha \gamma_p^{2/3} f^{1/2} r^{1/2}}{\mu^{1/2} b^2}$$

$$\sigma_c \approx \frac{\mu b \sqrt{f}}{r}$$

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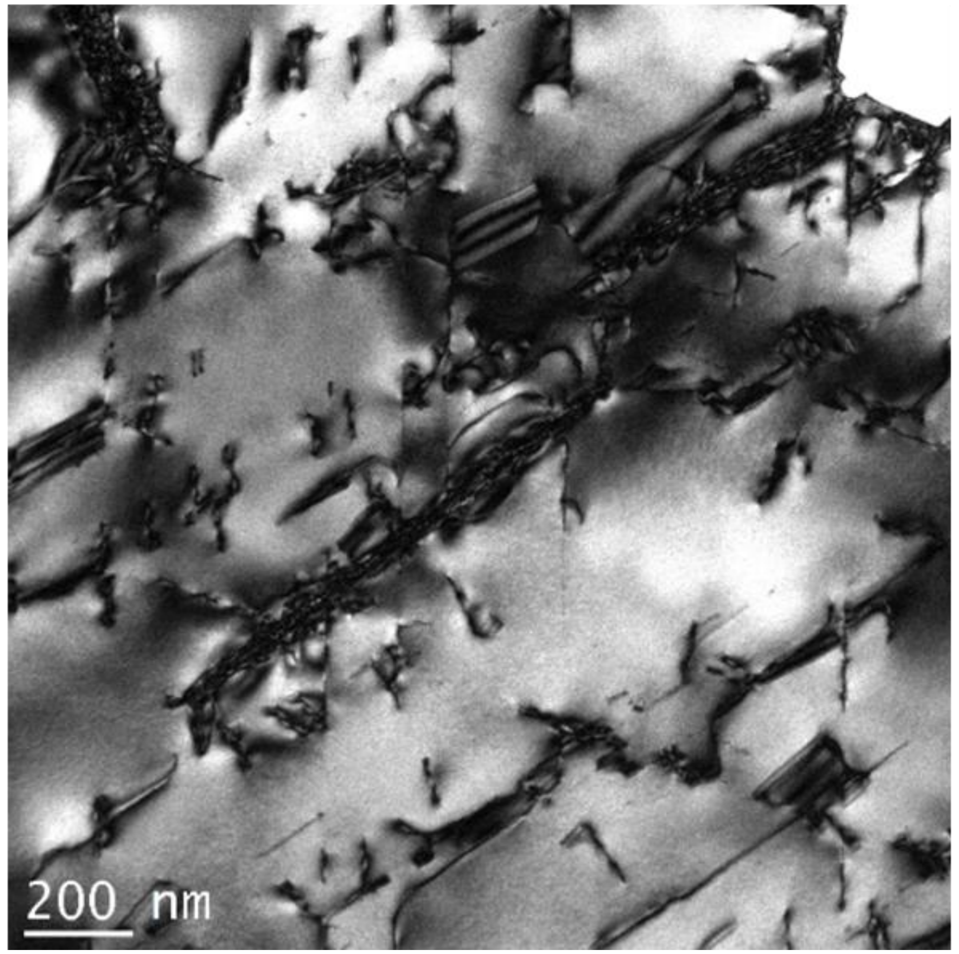
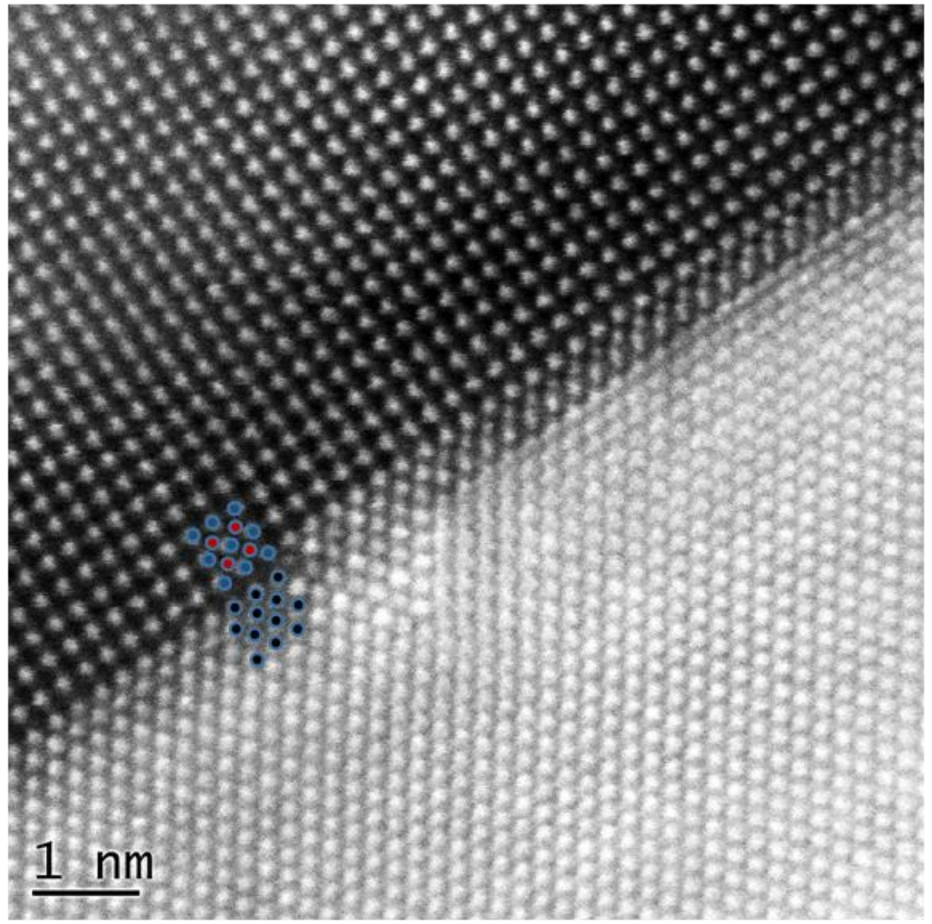
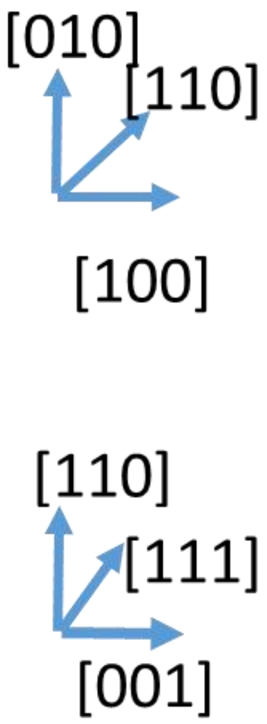
Coherent NiBe precipitates in CoCr superalloys

Brightfield TEM

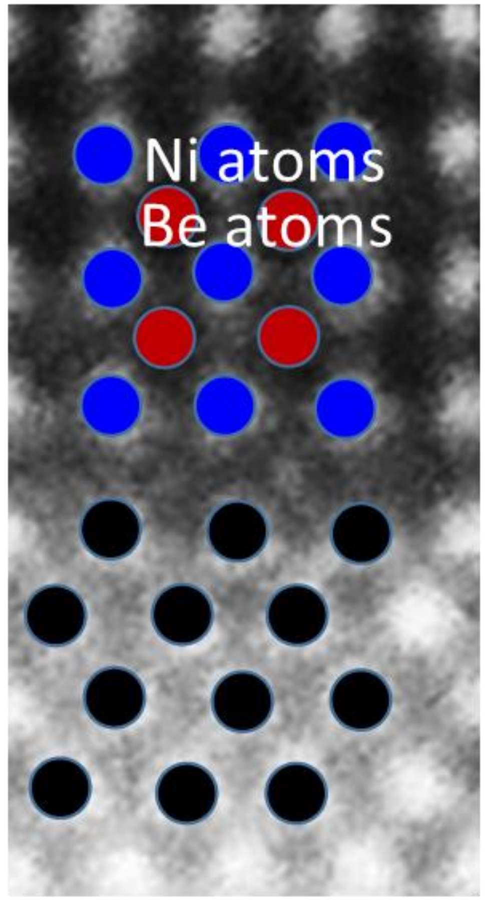


HR-STEM HAADF





Atomic Structure of the interface



NiBe B2 ordered intermetallic [100] Z.A.

Be atomic columns being low Z are barely visible

CoCr fcc alloy Solid solution with limited ordering [110] Z.A.

Bain Orientation {100}NiBe||{100}CoCr and <010>NiBe||<110>CoCr

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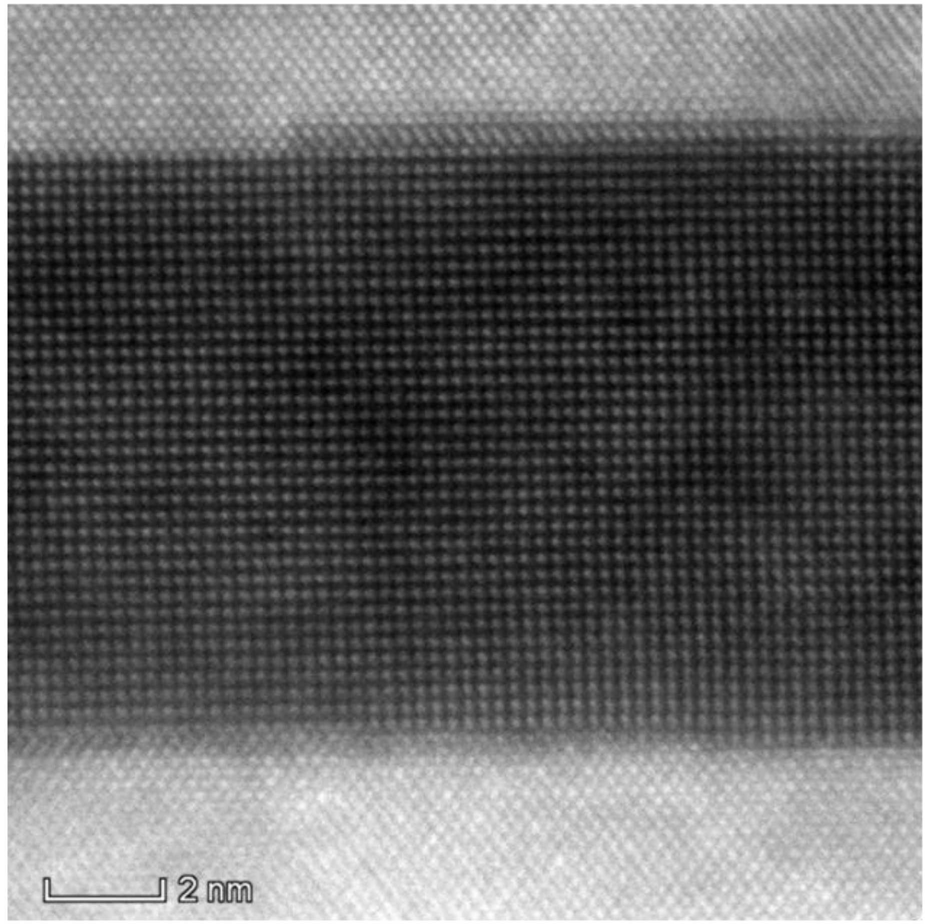
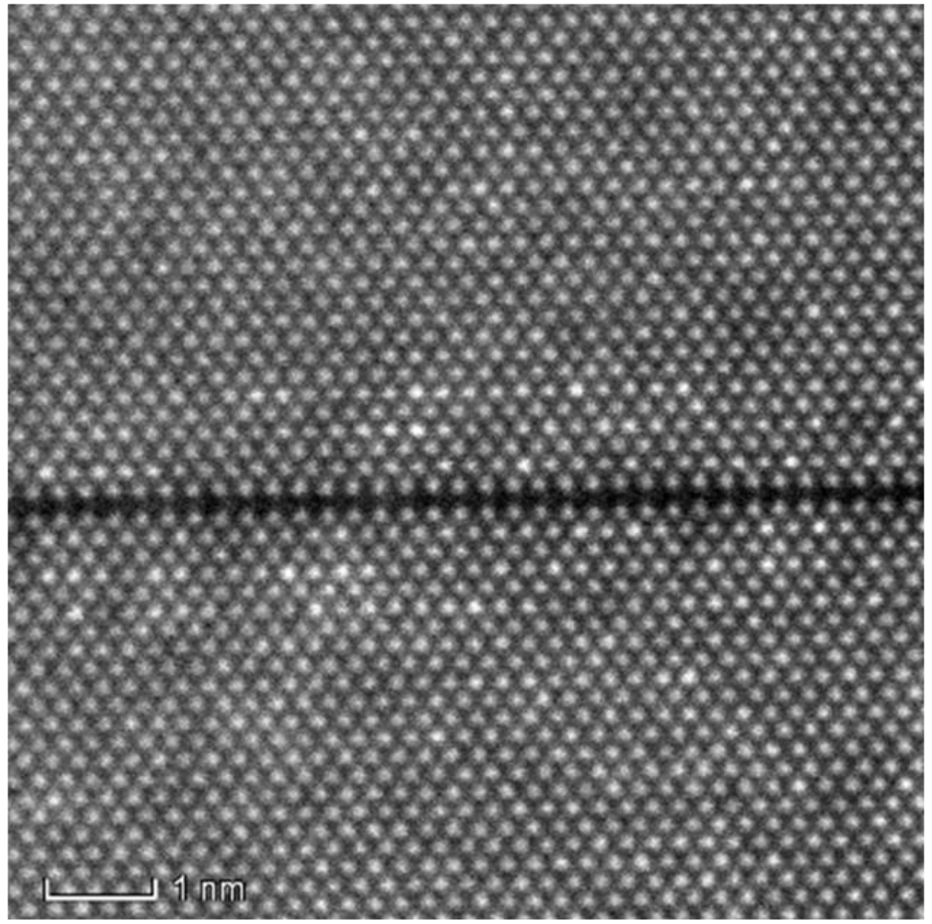
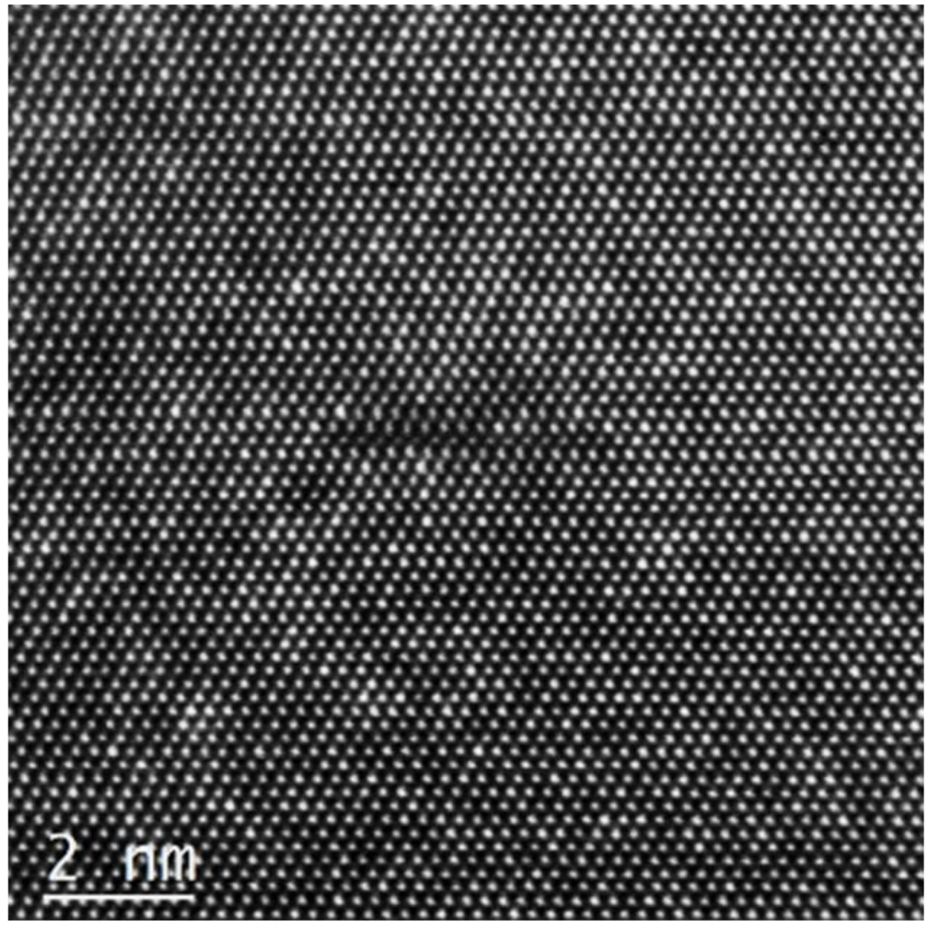
Aging of NiBe precipitates in CoCr superalloys

Semi-coherent precipitate	Coherent Be GP zone	Be Interstitial cluster
		
Bypass-cutting mechanism	Cutting mechanism	Defect Pinning/ dragging and increased Peierls stress

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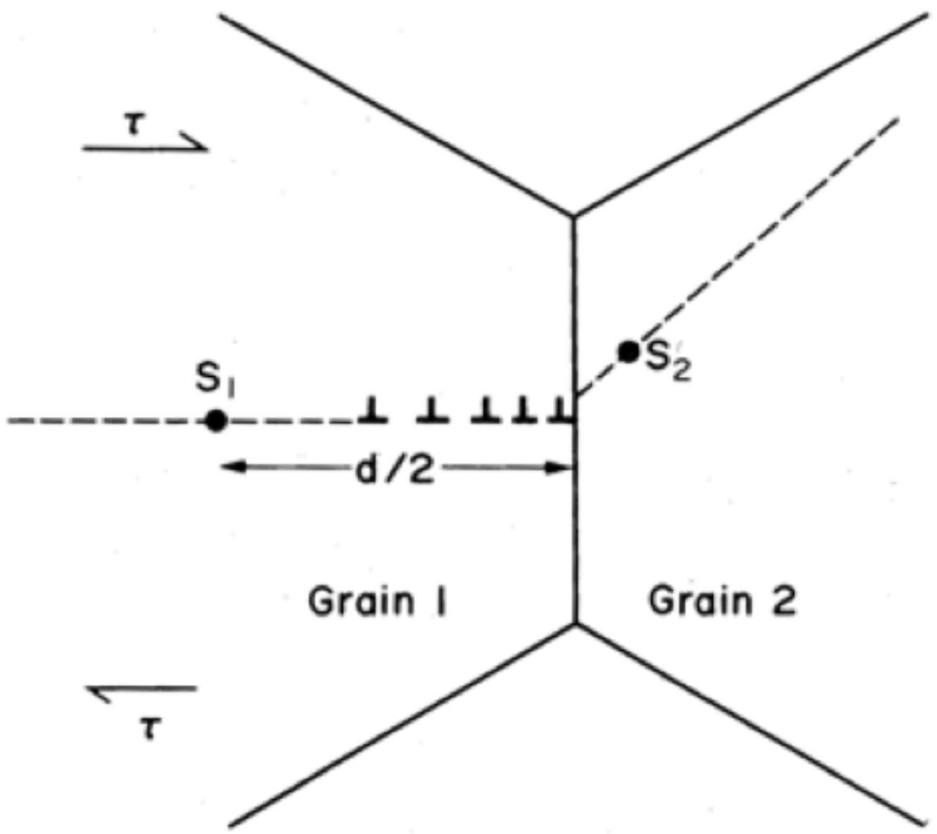
Hall-Petch law

$$\sigma_y = \sigma_0 + \frac{k}{\sqrt{d}}$$

Model based on the stacking of dislocations at grain boundaries



Stress applied on the shear-stress plane: τ

Stress on the leading dislocation: τ_1

$$\tau_1 = n\tau$$

$$\tau = \frac{\frac{\mu b^2}{2\pi(d/2)}}{n} = n\frac{\mu b^2}{\pi d} = n\frac{A}{d}$$

$$\tau_1 = n\tau = n^2\frac{A}{d} = \frac{d}{A}\tau^2$$

Critical stress $\tau_1^* = \frac{d}{A}\tau^2 \Rightarrow \tau = \sqrt{\tau_1^*\frac{A}{d}} \Rightarrow \sigma_y = \frac{k}{\sqrt{d}}$ $\qquad k = \frac{\sqrt{\tau_1^* A}}{m}$ Schmid factor

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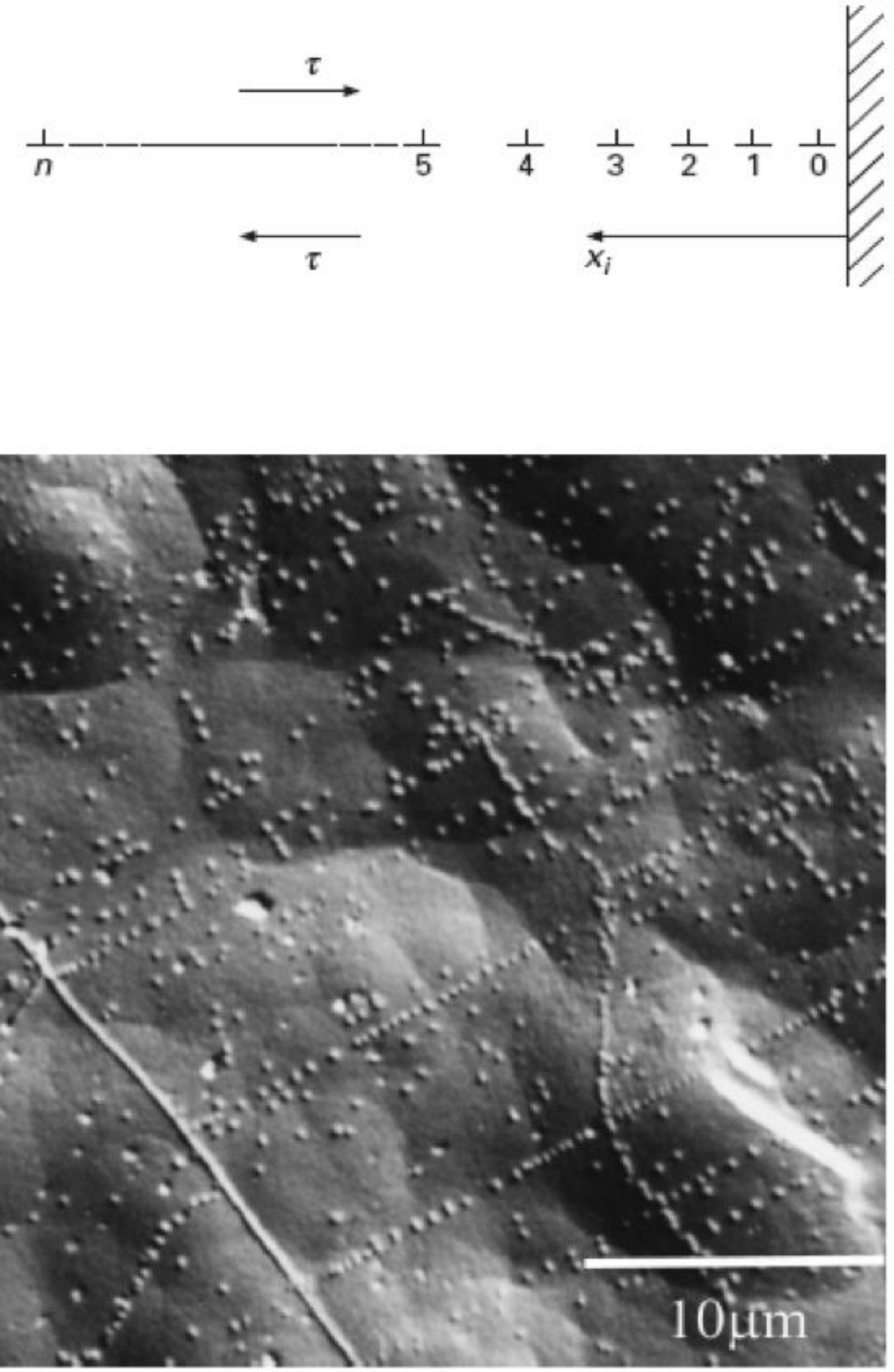
Hall-Petch law



$$\sigma_y = \sigma_0 + \frac{k}{\sqrt{d}}$$

Pile-up of dislocations in Cu observed by etch-pitting

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